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Task 8523  
TRACOR Project 035 001 01  
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THIRD QUARTERLY TECHNICAL PROGRESS REPORT

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THIRD QUARTERLY TECHNICAL PROGRESS REPORT

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Approved by:

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J. S. Dow

Prepared by:

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H. A. Reeder

Project Director

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## 1.0 INTRODUCTION AND SUMMARY

### 1.1 Introduction

The purpose of the present contract is to extend SLR tracking and detection concepts to simultaneous processing of multiple active sonar receiver outputs. In particular, the effect under the present contract assumes a sonar system consisting of both low-Doppler (FM replica correlator) and high-Doppler (CW) receivers. The previously developed SLR tracking and detection algorithm which employed the dimensions of range and bearing has been generalized to include the Doppler dimension information which is available at the output of the high-Doppler receiver. This expansion provides the necessary groundwork for extending the SLR processing technique to multiple sonar receiver outputs.

Under prior exploratory development contracts TRACOR developed and evaluated the SLR detection technique for a low-Doppler receiver (linear FM slide coded replica correlator) in which only range and bearing were used as tracking dimensions. The first major task in the present contract was therefore the extension of the SLR technique to provide a tracking algorithm for use at the output of a high-Doppler receiver\* where the additional dimension, namely Doppler frequency, is available. There are two possible means by which this can be done. The first of these methods requires significantly more computer

*low doppler receiver (linear FM slide coded replica correlator.)*

---

\*The high-Doppler receiver considered here consists of a comb filter bank whose parallel outputs are detected and then (possibly) OR-gated. The signal waveform is a long CW pulse whose duration is such that the echo spectrum and the reverberation spectrum are separated for high speed targets. This permits longer range target detection than would be possible if these spectra overlapped.



storage capacity than the other, but also offers better performance. Hence, a comparison of the performance and computer loading for these two approaches is required before an objective choice can be made between the two.

Both approaches utilize Doppler shift as a dimension in addition to the range and bearing dimensions. In the first approach, the input to the SLR algorithm consists of the parallel output time series from each channel of the Doppler filter bank. Such a multiplicity of outputs will exist on each preformed beam. Thus, if there are  $K$  preformed beams,  $M$  range resolution cells and  $N$  Doppler frequency channels, there will be  $K \cdot M \cdot N$  data samples to process on each ping cycle.

In this three dimensional space, a ping-to-ping track can exist only if the dynamics in the three dimensions are consistent with physically realizable target motion. When such ping-to-ping track consistency is found to exist, a linkage is formed by the process of joint log likelihood ratio formation and prediction of the future position of the target on the next ping cycle. This alternative is referred to as the multichannel high-Doppler SLR algorithm since all of the comb filter bank channels are used for linkage formation and tracking.

The second alternative retains only the information emerging from the comb filter bank after OR-gating or selection of the largest filter output amplitude in a given range resolution cell and on a given beam. The Doppler shift and amplitude of the largest output of the comb filter bank are then used to form linkages for tracking purposes. The track is still carried out in the three dimensional space, but there are fewer samples to be considered for linkage since only a single output is taken per range resolution cell per beam. This alternative is referred to as the OR-gated single output high-Doppler SLR algorithm. This





approach is desirable from a computer loading viewpoint because of the reduced data rate given by the OR-gate. However, the process of OR-gating causes some processing losses which are not experienced in the multichannel high-Doppler SLR algorithm.

## 1.2 Summary of Results

During this reporting period both the multichannel and OR-gated high-Doppler SLR algorithms were tested for comparative performance. The details of the test plan and presentation of test data are given in Appendix A.

The test results indicated that the performance (average track intensity)\* of the OR-gated high-Doppler SLR processor is essentially the same as the multichannel high-Doppler SLR processor. An example of this is shown by the circles and squares in Fig. 1. Other examples are given in Appendix A. However, the computer loading of the OR-gated high-Doppler SLR processor was only 36% of the computer loading of the multichannel high-Doppler SLR. Because of this significant saving in computer loading (and indirectly computer execution time) the OR-gated high-Doppler SLR processor will be adopted for future use.

A low-Doppler SLR processor based on the chi-square, four degree of freedom model for fluctuating targets\*\* was implemented and tested. The details of the implementation and the presentation of test data are given in Appendix B. The tests indicate that for equal average signal-to-noise ratios at the

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\*The criterion of performance used here is average target track contrast, i.e., the average target intensity divided by the average background clutter intensity. Since thresholds are adjusted to maintain constant average clutter density and intensity, this contrast is given directly by the average track intensity.

\*\*This model is based on sea test data as reported in the Second Quarterly Progress Report.

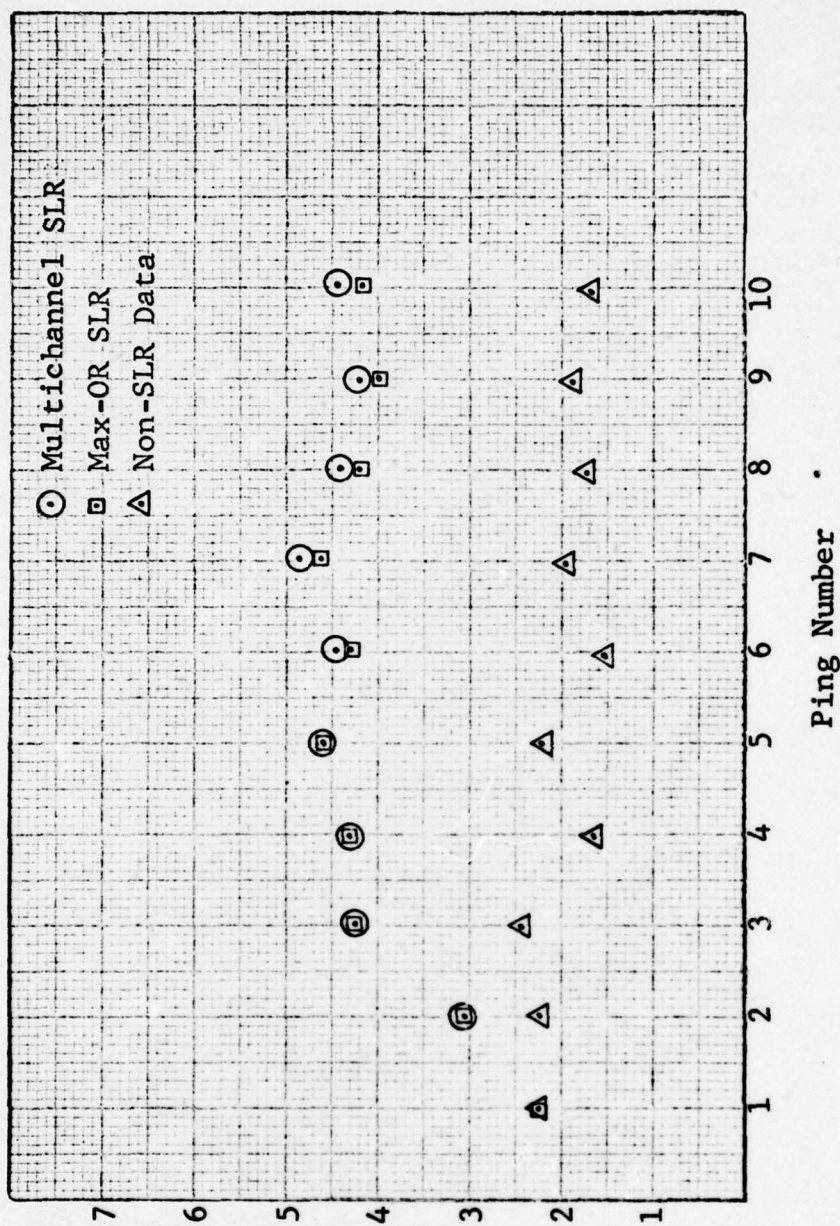


FIG. 1 - AVERAGE TRACK INTENSITY FOR MULTICHANNEL SLR, MAX-OR SLR, AND NON-SLR SIGNAL-PLUS-NOISE TRACK S/N = 3.2 dB, DESIGN S/N = 6.2 dB (U)



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input to the SLR processors the low-Doppler SLR processor has essentially the same performance as the SLR processor based on Rayleigh-Rice statistics that was used in previous implementations. The computer loading was also approximately the same.

As an interesting variation, the chi-square four degree of freedom model data were processed by an SLR processor designed for Rayleigh-Rice statistics. The performance results were nearly the same as when Rayleigh-Rice data were processed. This is a reasonable result since the processor based on Rayleigh-Rice statistics is similar to a linear detector<sup>\*</sup> whereas the chi-square, four degree of freedom processor is similar to a square law detector (Appendix B). It has been shown<sup>\*\*</sup> that the performance of linear detectors and square law detectors are almost the same; hence, it is reasonable to expect that the two SLR processors will generate essentially the same performance when driven by similar input data. This test shows that the readily computed Rayleigh-Rice statistics may be used in place of the more involved log likelihood ratio for the chi-square, four degree of freedom, if desired.

As reported in the Second Quarterly Progress Report, a mathematical model which permits the calculation of the probability of display clutter and probability of detection at the output of the SLR processor has been developed. A more detailed description of this model is given in Appendix C. This model was used to

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<sup>\*</sup>H. A. Reeder, "Computer Utilization of Sequential Hypothesis Testing for Detection and Classification of Sonar Signals," TRACOR Document 67-717-U, 7 December 1967.

<sup>\*\*</sup>P. B. Brown, "A Comparison of the Performance of Several Signal Processors," TRACOR Document 66-203-U, 10 March 1966.





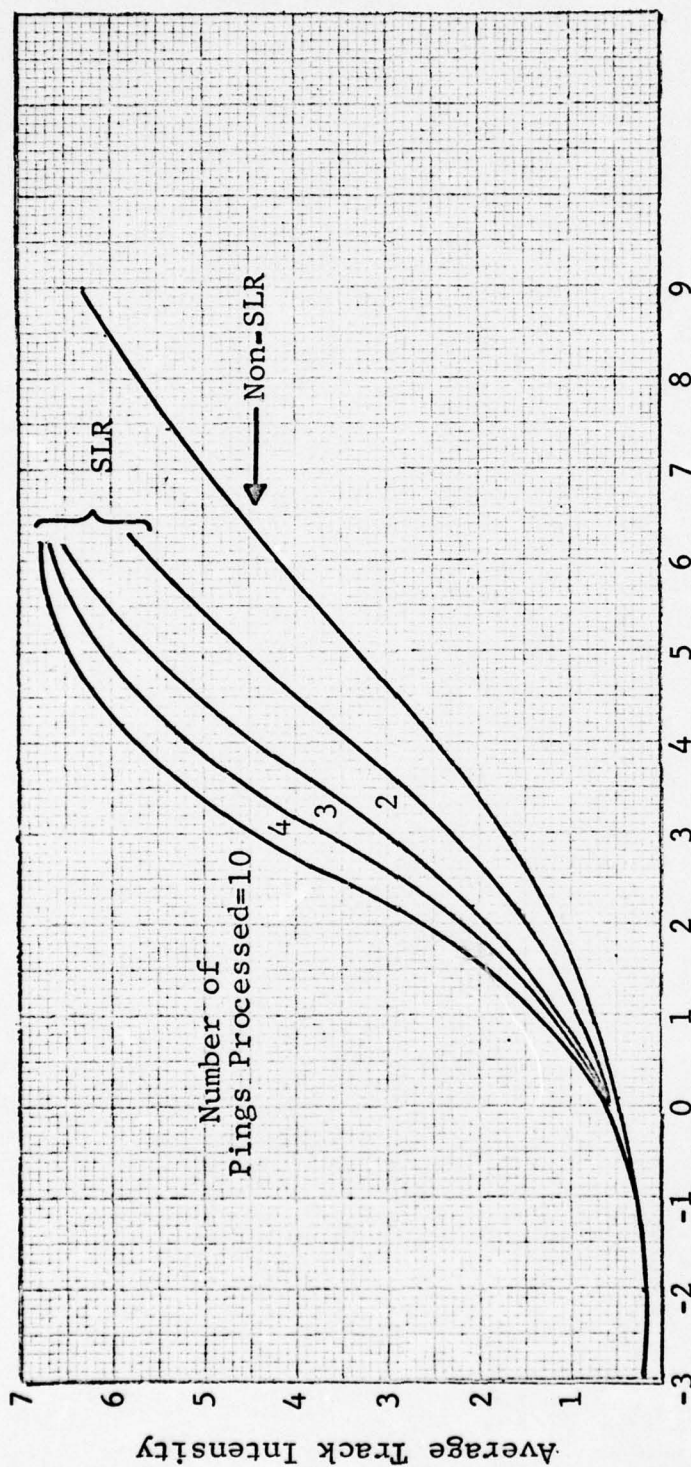
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calculate expected average intensities for a variety of input conditions. The results of these calculations are presented with the observed test data in Appendices A, B, and C. The predicted average track intensities agreed well with the observed data.

The use of this model permits easy (and inexpensive) calculation of the expected performance of the SLR processor as a function of the input signal-to-noise ratio and other parameters. As an example of the use of this model, Figs. 2 and 3 present the calculated average track intensities for signal-plus-noise tracks that have been processed for each of several ping cycles. Figure 2 is for the low-Doppler SLR and Fig. 3 is for the high-Doppler SLR. Curves are parametrically dependent on the number of pings over which the SLR processor has been operating. Also shown is the average intensity for non-SLR processed tracks. In each case the signal-to-noise ratio is measured just prior to envelope detection but after correlation (low-Doppler case) or contiguous filtering (high-Doppler case).

The sets of curves reveal some interesting features of the SLR algorithm. First, for any given input signal-to-noise ratio the track average intensity increases rapidly for the first three or four ping cycles then more slowly until a near steady state condition is reached after 10 pings. This steady state condition exists because of two factors. One, the SLR has a high internal threshold that limits the maximum value of the joint log likelihood ratio of a track. This prevents a strong track from linking with insignificant noise samples to form long lasting (though decaying) false tracks. Also, it will keep the track statistics within a given dynamic range when the SLR is implemented on a small digital computer with a short word length. This upper bound on the log likelihood ratio limits the growth of the track statistic.

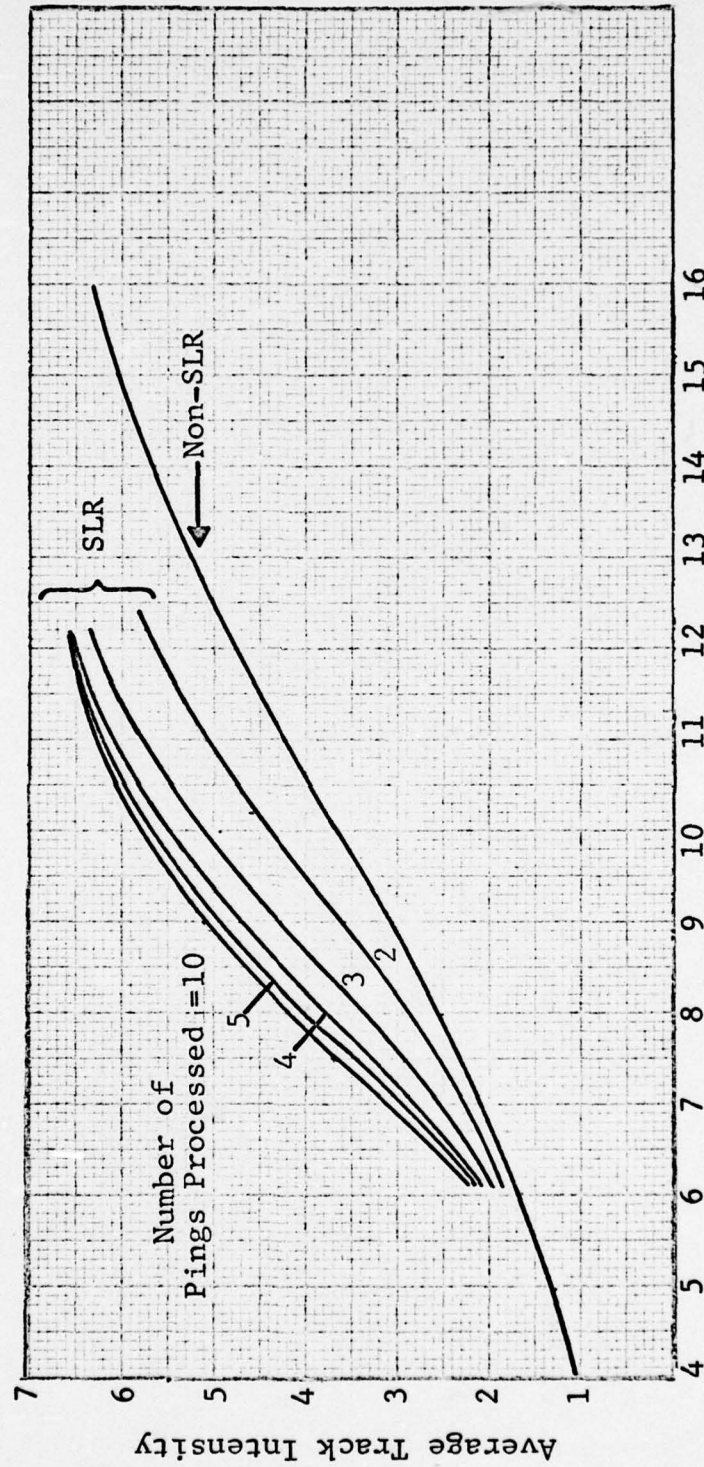




Signal-To-Noise Ratio (Measured in the Resolution Bandwidth), dB

FIG. 2 - AVERAGE TRACK INTENSITY FOR HIGH-DOPPLER SLR AS A FUNCTION OF INPUT

SIGNAL-TO-NOISE RATIO (U)



Signal-To-Noise Ratio (At Correlator Output), dB

FIG. 3 - AVERAGE TRACK INTENSITY FOR LOW-DOPPLER SLR AS A FUNCTION OF INPUT SIGNAL-TO-NOISE RATIO (U)



Second, it is clear that for small signal-to-noise ratios the gain of the SLR is quite limited. This holds because of the use of an initial thresholding of the input data. This input threshold is designed to eliminate the small amplitude samples that are mostly noise in order to reduce computer loading and processing time. However, there is a probability that some signal-plus-noise samples will also be eliminated. These missed track samples will degrade the track statistic. The effect of these missed signal-plus-noise samples is more pronounced for low signal-to-noise ratios and after several pings cycles. In each case the probability of at least one or more missed track sample is significant.\* The figures show that for modest signal-to-noise ratios the improvement in the average track intensity can be a dramatic 2.5 to 3.5 intensity levels. On the other hand, the performance for lower signal-to-noise ratios shows little improvement. This phenomenon has been noted before.\*\* It is caused by both the missed track samples and the similarities between the noise and signal-plus-noise densities for these signal-to-noise ratios. Appendix C also discusses the effects on performance of varying other parameters such as initial threshold and window widths for the high-Doppler SLR processor.

It is important to note two facts in regard to this behavior of the SLR processor. First, the present SLR processing algorithm is designed with reasonable computer capacities in mind. This requires that such design features as initial thresholding be employed. When the shipboard computer capacity increases,

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\*For a further discussion of this, see Section 4.3.3 of H. A. Reeder, "Reduction of Computer Requirements for the Sequential Likelihood Ratio Processor," TRACOR Document T70-AU-7242-U, 29 July 1970.

\*\*H. A. Reeder (1967) op. cit. and H. A. Reeder (1970) op. cit.





then some of these constraints can be relaxed so that gains with lower signal-to-noise ratios can be realized. The point is that the potential is there and yet unrealized because of the constraints of computer capacity. Tracking in accuracy is another contributor to reduced gain since uncertainties in target signal location permit noise alone to be occasionally integrated into the track. To demonstrate what the ideal performance can be, three runs were made where first the initial threshold was set equal to zero; second, the tracking uncertainties were zero; and third, both of these conditions were assumed. These results are shown in Figs. 4, 5, and 6 where it is clear that greater gains are produced. In Fig. 6 we see that for an intensity level of 4, the SLR produces (over a non-SLR system) a gain in terms of input signal-to-noise ratio of 2.1 dB in 2 pings, 3.3 dB in 3 pings, 4.1 dB in 4 pings, 4.5 dB in 5 pings and 5.3 dB in 10 pings.

Another point is this. "Processing gain" is but a part of what the SLR processor offers. An equally important feature of the SLR processor is its ability to process large quantities of data over long periods of time with consistency and predictability in much the same way as an untiring, alerted operator.

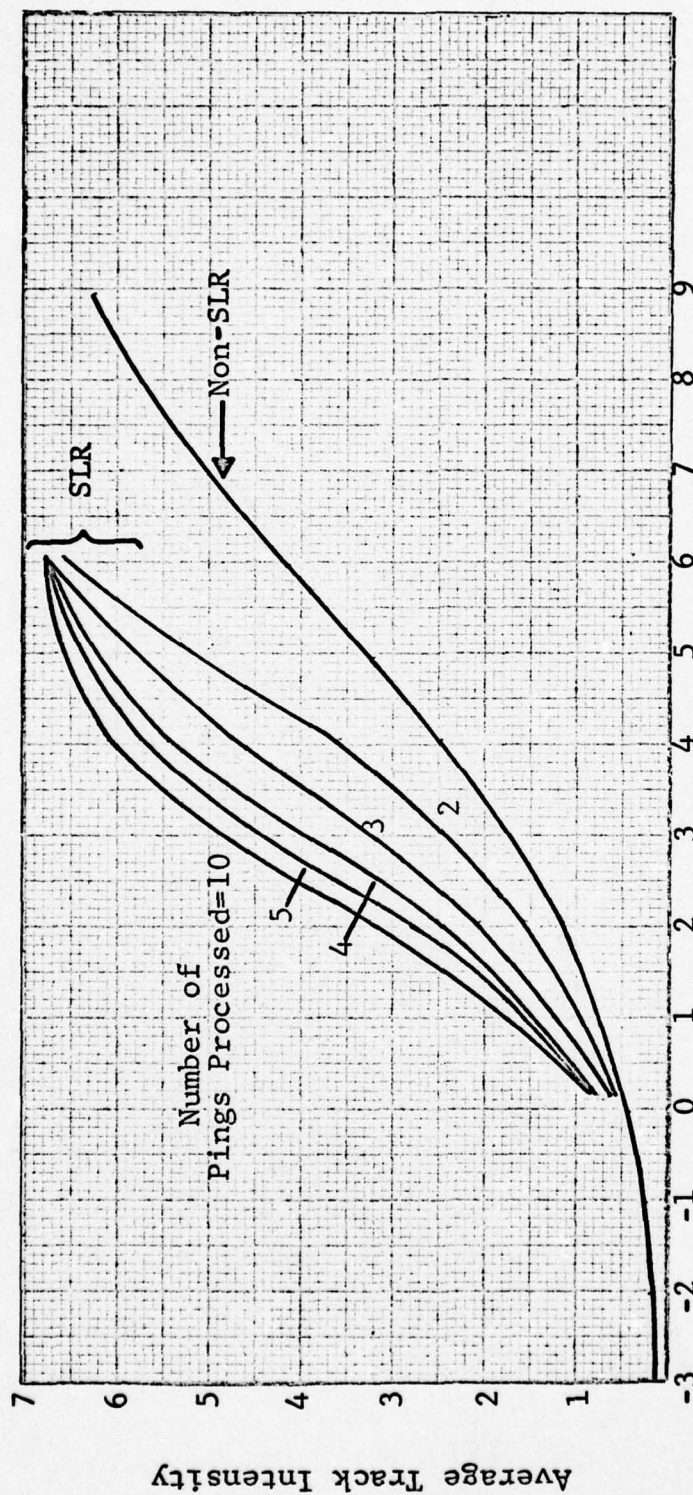
## 2.0 ACTION ITEMS ACCOMPLISHED

A. Completion of tests of multichannel high-Doppler and OR-gated high-Doppler SLR algorithms (Milestones 8, 10).

B. Modify and test of low-Doppler SLR algorithm (Milestones 11, 12, 13, 14).

C. Start development of combined high- and low-Doppler SLR tracking algorithm (Milestone 15).

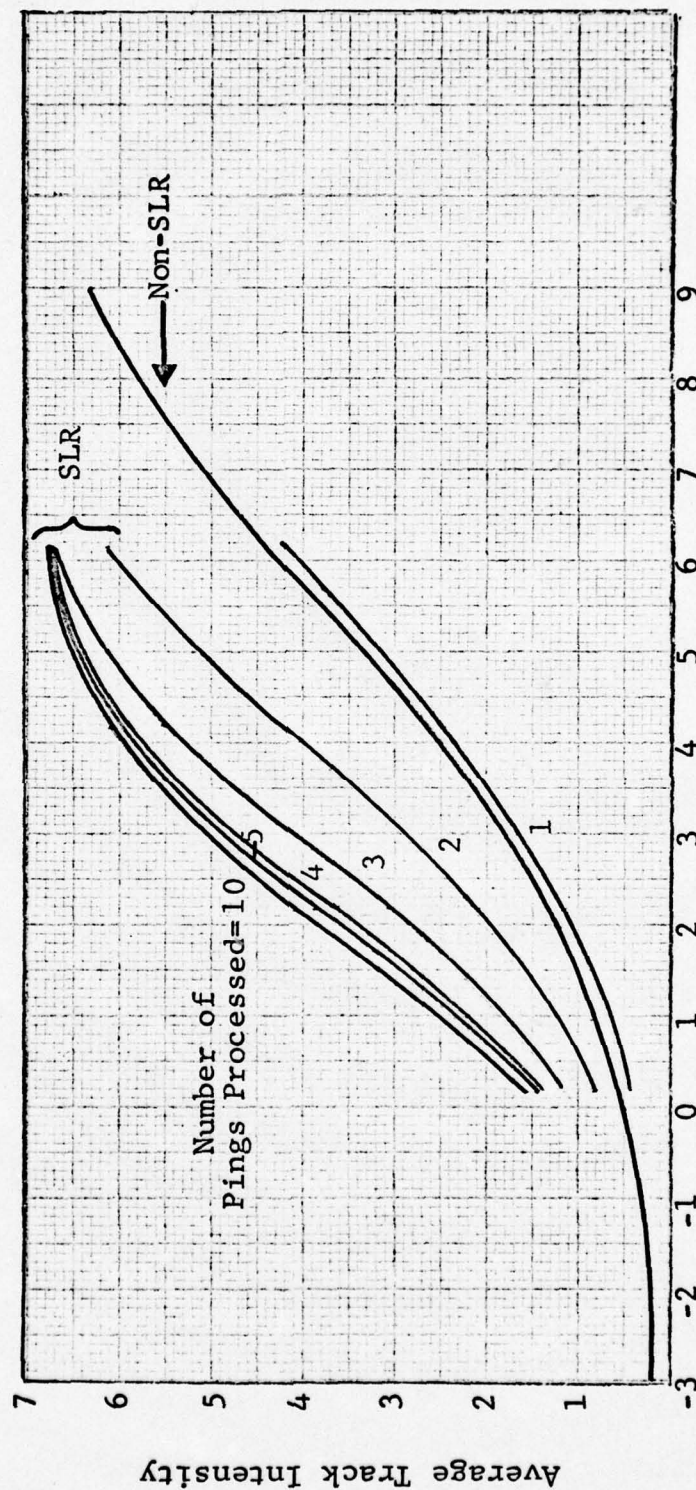




Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. 4 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN S/N = 6.2 dB, NO INITIAL THRESHOLD (U)



Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. 5 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN  $S/N = 6.2$  dB, NO UNCERTAINTY IN TARGET LOCATION (U)



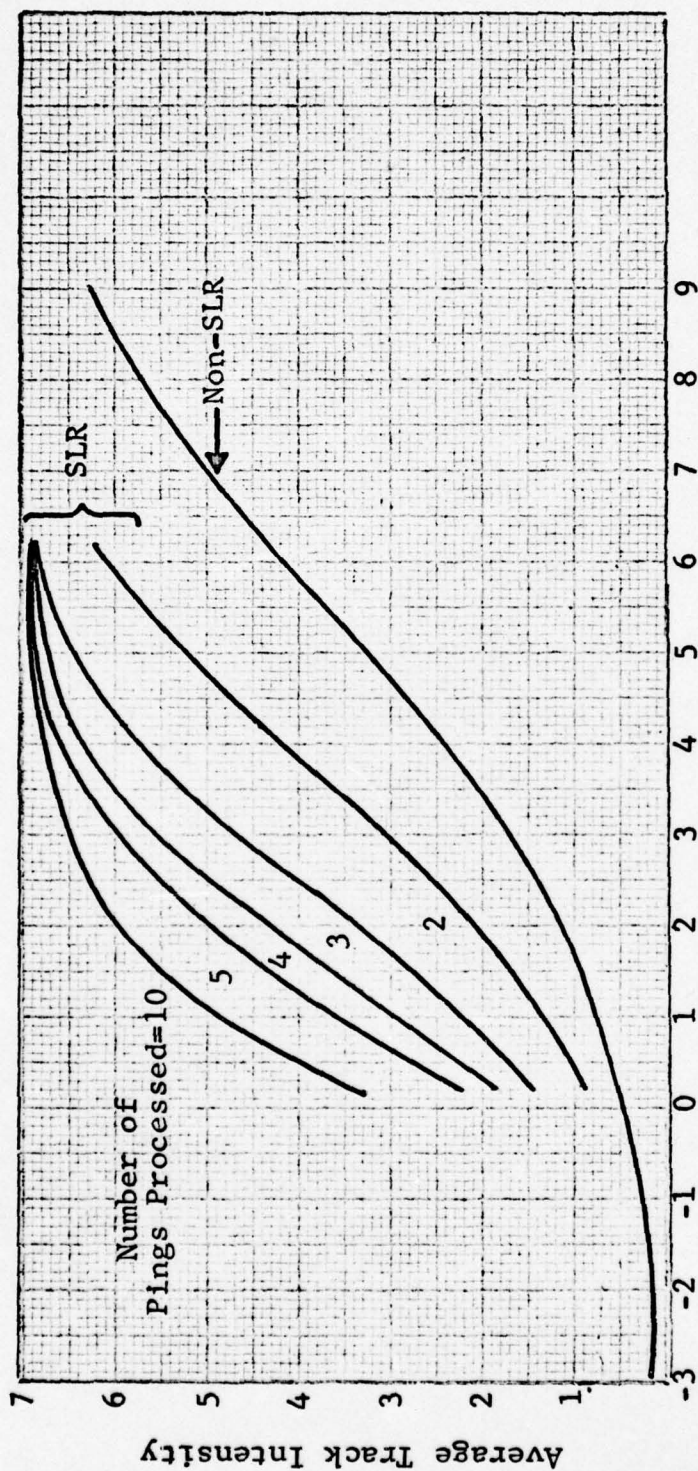


FIG. 6 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN  $S/N = 6.2$  dB, NO INITIAL THRESHOLD,

NO UNCERTAINTY IN TARGET LOCATION (U)



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### 3.0 ACTION PLANNED NEXT REPORTING PERIOD

A. Complete the development of combined high- and low-Doppler SLR tracking algorithm and test (Milestones 16, 17, 18, 19, 20).

B. Develop a general framework for the inclusion of additional sonar receiver outputs in SLR processing (Milestones 21, 22).

C. Develop SLR algorithm to process data from ARL's digital sonar system.

### 4.0 MILESTONES AND TASK COMPLETION DATES

Milestones 8, 10, 11, 12, 13, 14, and 15 completed.

### 5.0 PUBLICATIONS/REPORTS

None.

### 6.0 TRAVEL

<u>Date</u>	<u>Personnel Visited</u>	<u>Purpose</u>	<u>TRACOR Visitors</u>
27 Oct 70	K. Buske D. Stricker	Discuss project status	J. Dow J. Bardin H. Reeder





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7.0 PROBLEM AREAS

None.

8.0 REQUIRED NAVSHIPS ACTION

None.

9.0 SUPPORTING DOCUMENTATION/TRIP REPORTS

Appendix A - "Test Results from the Study of Two Versions of the High-Doppler SLR Algorithm," H. A. Reeder.

Appendix B - "Test Results from the Study of the Low-Doppler SLR Algorithm," H. A. Reeder.

Appendix C - "Description of the Model to Predict Clutter and Detection Probabilities in the SLR Algorithm," H. A. Reeder.



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## APPENDIX A

### TEST RESULTS FROM THE STUDY OF TWO VERSIONS OF THE HIGH-DOPPLER SLR ALGORITHM



## A.1 INTRODUCTION

A major task in the present contract was to extend the SLR technique to provide a tracking algorithm for use at the output of a high-Doppler receiver (long CW-comb filter bank) where the additional dimension, Doppler frequency, is available. There are two possible means by which this can be done. The first of these methods requires significantly more computer storage capacity than the other, but also offers potentially better performance. Hence, a comparison of the performance and computer loading for these two approaches is required before an objective choice can be made between the two.

Both approaches utilize Doppler shift as a dimension in addition to the range and bearing dimensions. In the first approach, the input to the SLR algorithm consists of all of the parallel output time series from the Doppler filter bank. Such a multiplicity of outputs will exist on each preformed beam. Thus, if there are  $K$  preformed beams,  $M$  range resolution cells per beam, and  $N$  Doppler frequency channels, there will be  $K \cdot M \cdot N$  data samples to process on each ping cycle.

In this three-dimensional space, a ping-to-ping track can exist only if the dynamics in the three dimensions are consistent with physically realizable target motion. When such ping-to-ping track consistency is found to exist, a linkage is formed by the process of joint log likelihood ratio formation and prediction of the future position of the target on the next ping cycle. This alternative is referred to as the multichannel high-Doppler SLR algorithm since all of the comb filter bank channels are used for linkage formation and tracking.





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The second alternative retains only the information emerging from the comb filter bank after OR-gating or selection of the largest filter output amplitude in a given range resolution cell. The Doppler shift and amplitude of the largest output of the comb filter bank are then used to form linkages for tracking purposes. The tracking is still carried out in the three dimensional space, but there are fewer samples to be considered for linkage since only a single output is taken per range resolution cell per beam. This alternative is referred to as the OR-gated single output high-Doppler SLR algorithm. This approach is desirable from a computer viewpoint because of the reduced data rate given by the OR-gate. However, the process of OR-gating causes some processing losses which are not experienced in the multichannel high-Doppler SLR algorithm.



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## A.2 TEST PROCEDURE

In order to judge the differences in the two processors, they were both implemented and subjected to certain tests.\* Random data (simulated outputs from each of K resolvable Doppler channels), representing noise alone waveforms, were generated by the computer. These envelope samples were distributed as a Rayleigh random variable that has been summed over five independent samples. A single channel of random data was generated to represent the target. Its statistical behavior was describable by a Rayleigh distribution with mean square  $S + N$ , corresponding to the signal-to-noise ratio  $S/N$ , and incorporating Sewerling's chi-square two degree of freedom model.\*\* In some cases the signal was moved from resolvable Doppler channel to another in simulation of a reasonable maneuver of a real target. In other tests zero range and bearing rates were used to allow more target tracks to be processed per run. This was done solely to obtain sufficient statistical accuracy in the results. Input signals were prepared for several different values of the ratio  $S/N$ .

The data described in the previous paragraph were processed by each of the proposed high-Doppler processors. The output data from each processor were a number of log likelihood ratio packets located at specific range, bearing, and Doppler frequency resolution cells. There was a set of such packets for each ratio  $S/N$  and for each processor.

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\*Appendix B, First Quarterly Progress Letter.

\*\*Second Quarterly Progress Report.



For fixed S/N, the output packet structure obtained from the first alternative processor and the output packet structure obtained from the second alternative processor were compared in terms of the signal brightness relative to the average noise brightness which would be produced on the display. Such a comparison must be done carefully because the contrast between the signal and the noise is of prime importance in the interaction of the operator and the display.

Although this method of average track intensities is not as definitive as an observer experiment, it is a considerably more economical means for obtaining relative performance estimates of several systems. The choice of thresholds used in calculating intensity ratios is illustrated in Figure A-1 and is the same method used previously.\* Typical curves of probability of exceeding threshold versus threshold are shown for (a) noise out of the CW processor, and (b) noise out of the high-Doppler SLR processor. Only one curve for the high-Doppler SLR processors is shown since it was found that the noise curves for each alternative exhibited no significant difference.

This is not surprising since clutter marks are caused by the largest log likelihood ratio associated with a range and bearing resolution cell and this particular log likelihood ratio probably arises from linkage of two or more large individual log likelihood ratios that were the maxima of the Doppler channels of comb filter bank. The probabilities of exceeding each chosen threshold are fixed to conform with a typical display on a modern sonar system. This results in an equal noise marking probability for each display level regardless of the processor under consideration.

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\*Reeder (1970) op. cit.



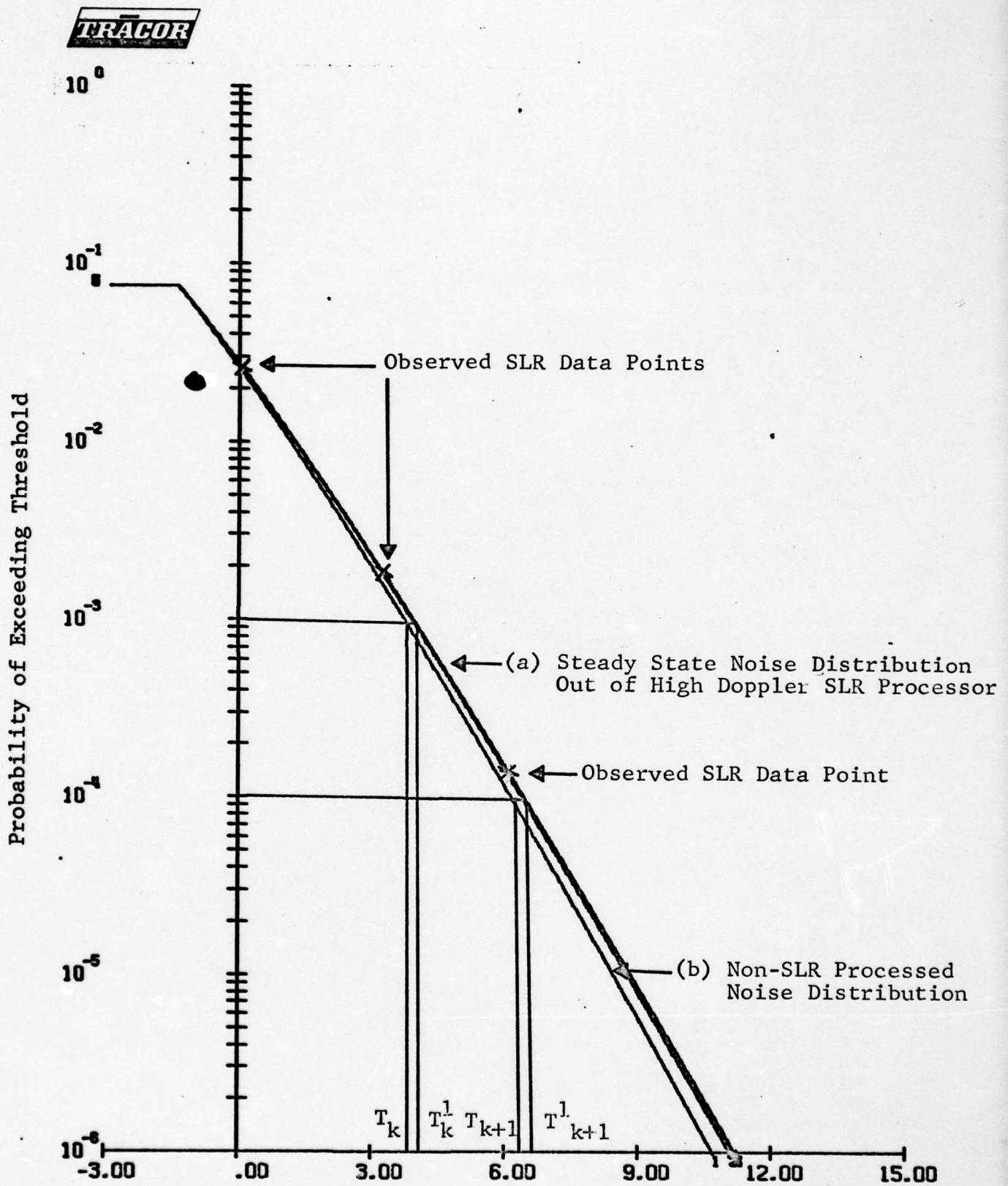


FIG. A-1 - THRESHOLD (UNITS OF LOG LIKELIHOOD RATIO) PROBABILITY OF EXCEEDING THRESHOLD VS THRESHOLD FOR NOISE ALONE. DESIGN S/N = 6.2 dB PRIMES DENOTE SLR THRESHOLD (U)



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A given waveform sample is subjected to the multiple display thresholding; if it lies between  $T_1$  and  $T_2$ , it is assigned the brightness 1, between  $T_2$  and  $T_3$ , brightness 2, etc. In the tests conducted, 40 signal-plus-noise tracks were considered for each ratio S/N, and the track brightness averaged over the 40 tracks. In order to economize all 40 tracks were run at one time. The tracks were well separated in range, bearing, and Doppler. If the 40 tracks had been allowed to move in a realistic fashion according to their Doppler, several tracks would have crossed and a confusing and unrealistic interaction would have occurred. In order to prevent this, the tracks were restricted to zero range and bearing rate. This does not restrict the validity of these tests so long as the actual range and bearing rate would be less than the maximum values allowed by the tracking windows. To confirm this, a limited test was performed in which free moving targets were used and no significant difference was found.



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### A.3 TEST RESULTS

The presentation of results consists of Figs. A-2 through A-5 that show the average track intensity for both high-Doppler SLR processor and non-SLR data for the first 10 ping cycles after the tracks start. Each figure is for a different value of the actual target signal-to-noise ratio  $S/N$ . This ratio is measured in the frequency band of a single comb filter and prior to envelope detection.

The figures show that little degradation occurs when the max-OR high-Doppler SLR processor is used. Figure A-4, one of the last cases calculated, shows only the max-OR processor results since it was felt that little would be gained by running the multichannel case. Figure A-6 shows the number of status units required while executing the two versions of the high-Doppler SLR processor. These data show that the max-OR high-Doppler SLR processor requires only about 36% of the number of status units that the multichannel SLR processor does. This means a large saving in required computer storage and also in execution time since not as many possible linkages have to be considered during each ping cycle. Since the max-OR high-Doppler SLR processor gives almost equivalent performance but considerably less computer loading, it is this alternative that should be adopted.



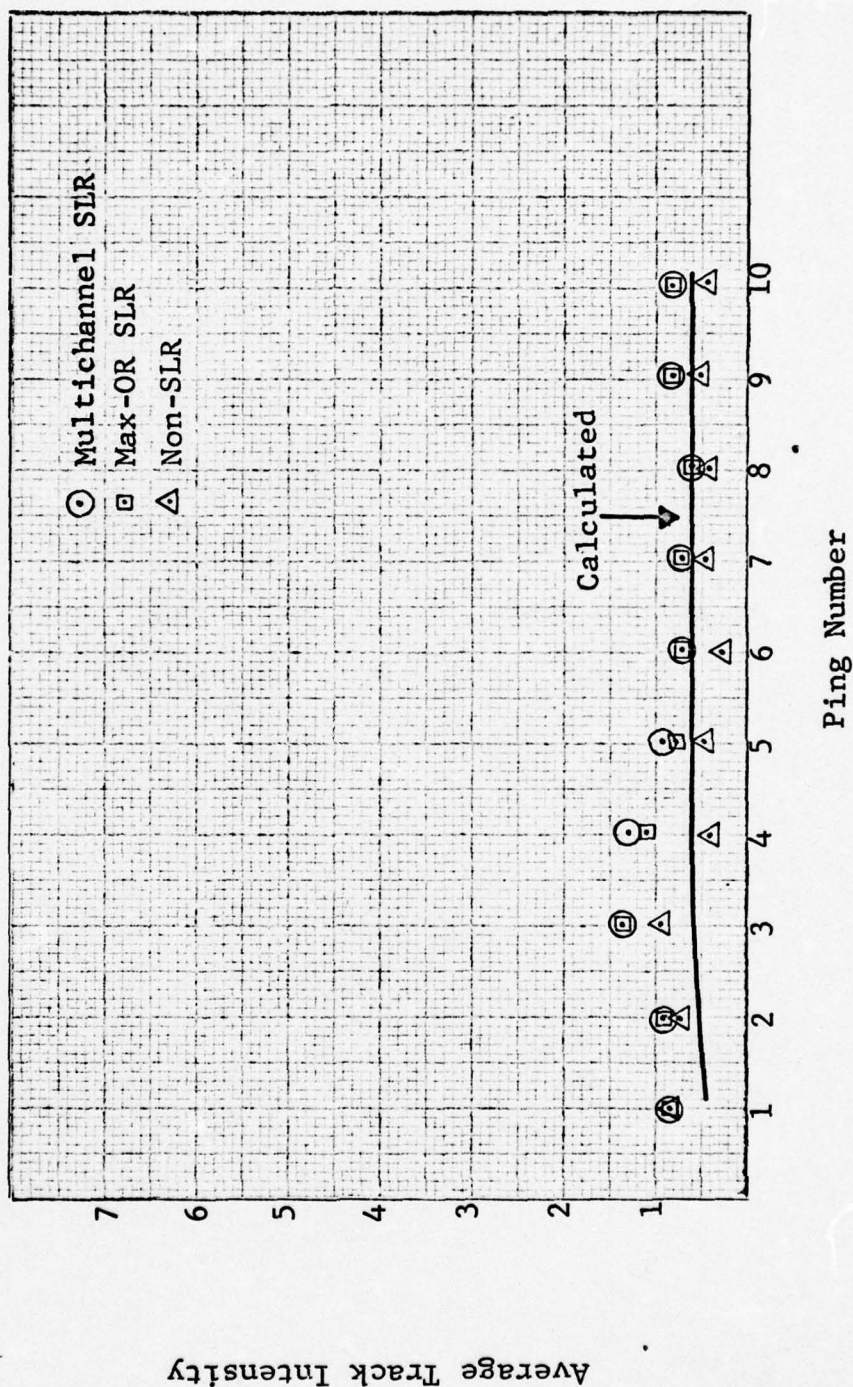


FIG. A-2 - AVERAGE TRACK INTENSITY FOR MULTICHANNEL SLR, MAX-OR SLR, AND NON-SLR SIGNAL-PLUS-NOISE TRACK S/N = 0.2 dB, DESIGN S/N = 6.2 dB (U)

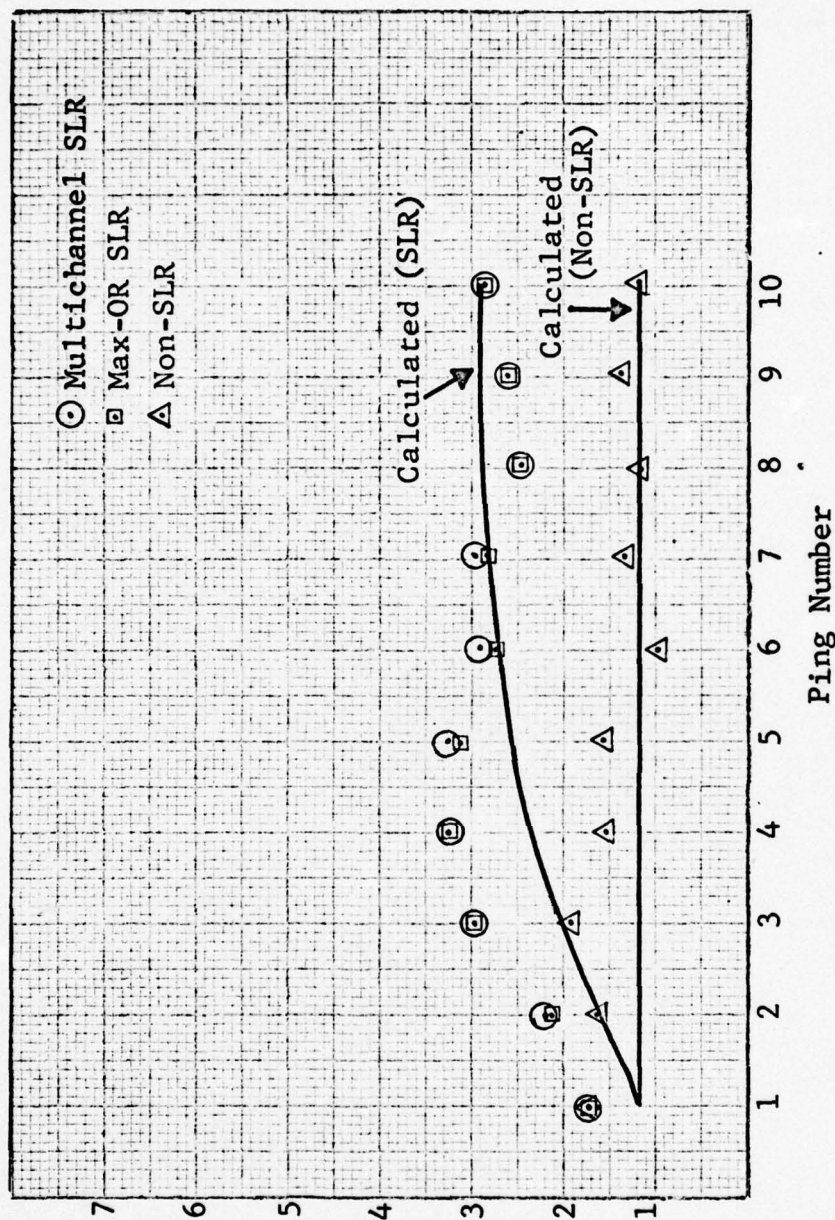


FIG. A-3 - AVERAGE TRACK INTENSITY FOR MULTICHANNEL SIR, MAX-OR SIR, AND NON-SLR SIGNAL-PLUS-NOISE TRACK S/N = 2.2 dB, DESIGN S/N = 6.2 dB (U)

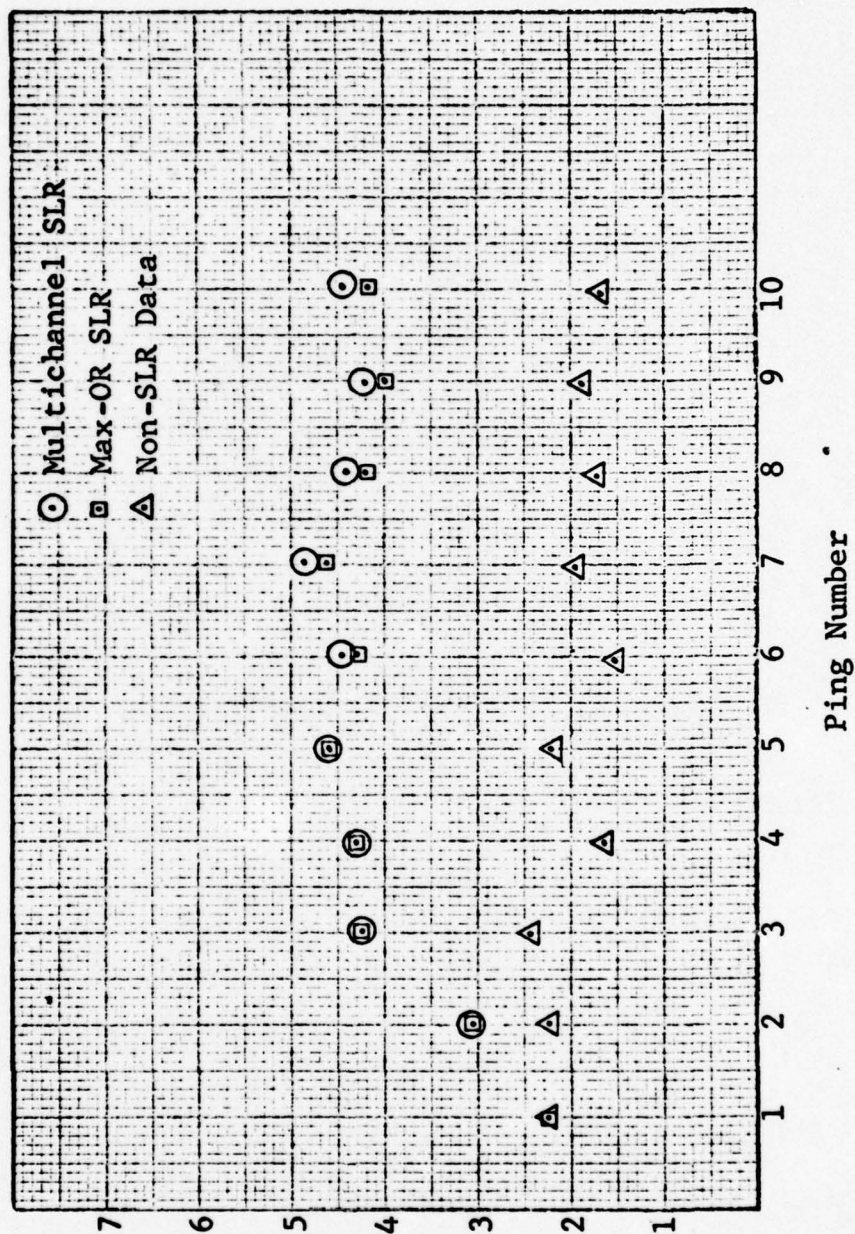


FIG. A-4 - AVERAGE TRACK INTENSITY FOR MULTICHANNEL SLR, MAX-OR SLR, AND NON-SLR SIGNAL-PLUS-NOISE TRACK S/N = 3.2 dB, DESIGN S/N = 6.2 dB (U)



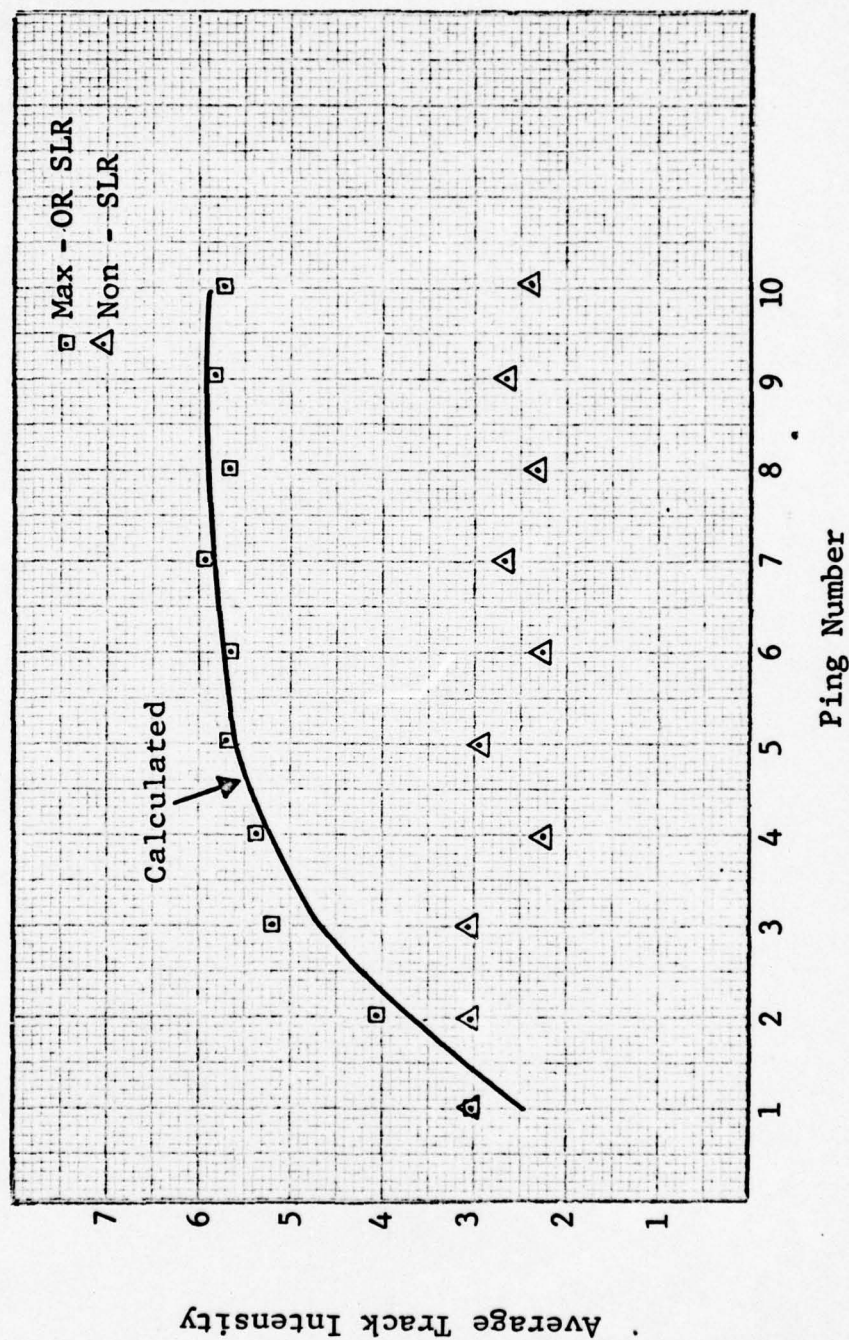


FIG. A-5 - AVERAGE TRACK INTENSITY FOR MAX-OR SLR AND NON-SLR SIGNAL-PLUS-NOISE  
DATA TRACK S/N = 4.2 dB, DESIGN S/N = 6.2 dB (U)

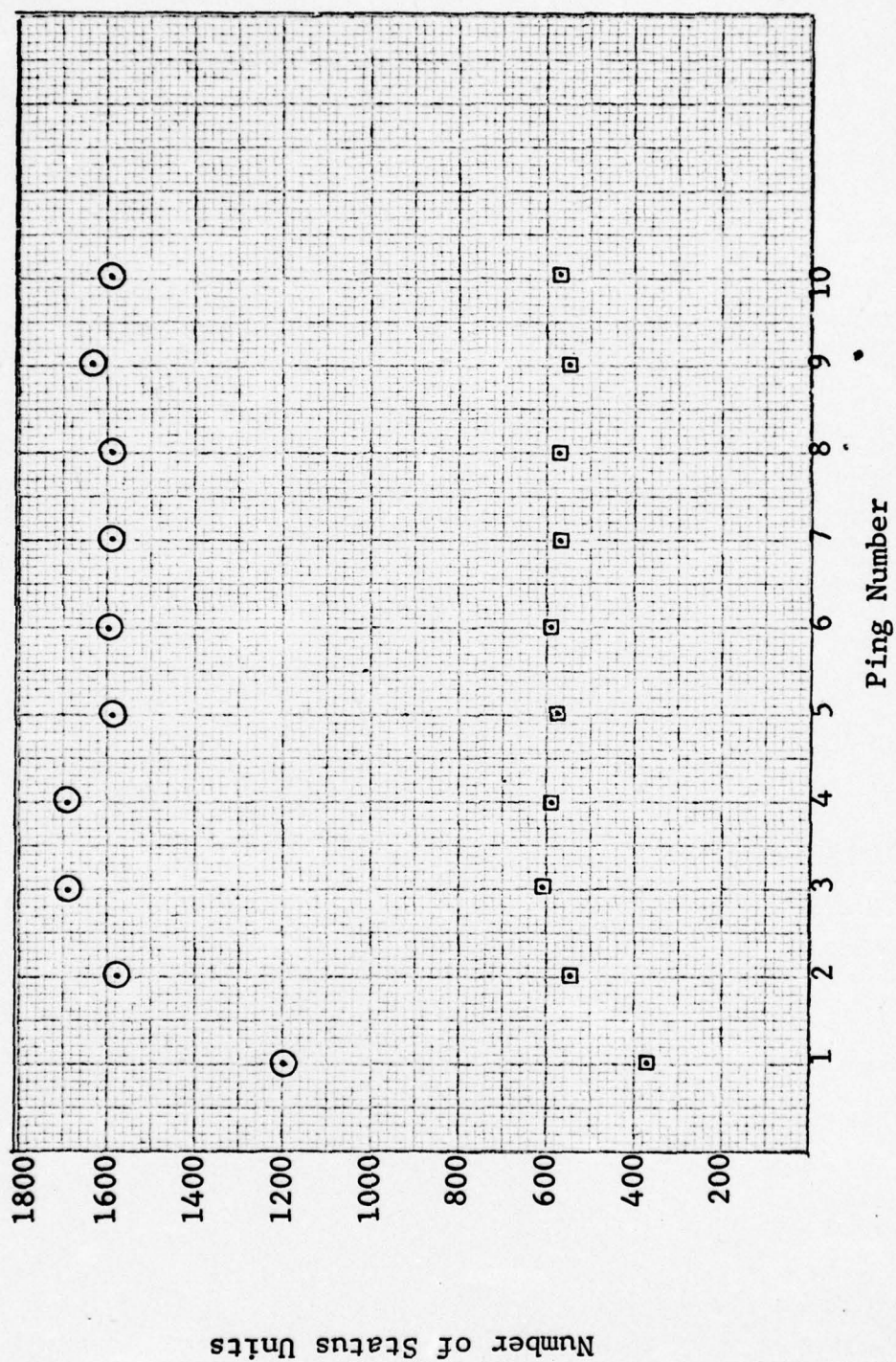


FIG. A-6 - COMPARISON OF NUMBER OF STATUS UNITS REQUIRED TO PROCESS NOISE ONLY  
DATA FOR MULTICHANNEL SLR (○) and MAX-OR SLR (□)  
DESIGN S/N = 6.2 dB (U)



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## APPENDIX B

TEST RESULTS FROM THE STUDY OF THE LOW-DOPPLER SLR ALGORITHM





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## B.1 INTRODUCTION

As previously reported\* a representative sample of target echoes gathered from AN/SQS-26 TECHVAL data was examined with regard to the statistical distribution of target echoes plus noise. Attempts were made to fit several fluctuating target models to the empirical distribution obtained from the '26 data. It was found that P. Sewerling's chi-square, four degree of freedom model\*\* for fluctuating targets provided the best description of the sample at the output of the coded pulse (CP) processor. A low-Doppler SLR processor using this model was implemented and tested for performance changes from the previous low-Doppler processor based on a non-fluctuating target model with Rayleigh-Rice statistics. No significant difference in track intensities or computer loading were found to exist.

---

\*Second Quarterly Progress Report.

\*\*R. S. Berkowitz, Modern Radar, John Wiley & Sons, New York, pp. 182-185, 1966.



## B.2 FORMULATION OF THE LOG LIKELIHOOD RATIO

In this section, the functional form of P. Sewerling's chi-square, four degree of freedom model density function and the likelihood ratio are described. An approximation to the log likelihood ratio is found by fitting, in the least square sense, linear function of the square of the envelope to the log likelihood ratio.

The probability density function for this model is given\* by

$$p(\chi/S+N) = \frac{4\chi}{(\alpha+2)^2} \left[ 1 + \frac{\alpha\chi^2}{2(\alpha+2)} \right] e^{-\frac{\chi^2}{\alpha+2}} \quad \chi \geq 0 ,$$

where  $\chi$  is the amplitude of the envelope,  
 $\alpha$  is the power signal-to-noise ratio.  
For noise alone  $\alpha = 0$

$$p(\chi/N) = \chi e^{-\frac{\chi^2}{2}} \quad \chi \geq 0 .$$

The likelihood ratio,  $\ell(\chi)$ , is

$$\ell(\chi) = \frac{p(\chi/S+N)}{p(\chi/N)} = \frac{4}{(\alpha+2)^2} \left[ 1 + \frac{\alpha\chi^2}{2(\alpha+2)} \right] e^{\frac{\alpha\chi^2}{2(\alpha+2)}} .$$

---

\*R. S. Berkowitz, op. cit.



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The log likelihood ratio,  $L(x)$ , is

$$L(x) = \ln l(x) = \ln \left( \frac{4}{(\alpha+2)^2} \right) + \ln \left( 1 + \frac{\alpha x^2}{2(\alpha+2)} \right) + \frac{\alpha x^2}{2(\alpha+2)} .$$

Note that the log likelihood ratio depends only on the square of the envelope. Since this is the case, a linear fit for the log likelihood ratio will be derived based on the square of the envelope. In order to simplify notation, let

$$c = \log \left( \frac{4}{(\alpha+2)^2} \right) ,$$

and

$$u = \frac{\alpha x^2}{2(\alpha+2)} .$$

Then

$$L(x) = c + \ln (1+u) + u .$$





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A reasonable criterion for fitting a curve is to minimize the mean square error between the actual curve and the proposed fitted curve (in this case,  $au + b$ ) over some interval of interest  $(T_1, T_2)$ . Let

$$M(a,b) = \int_{T_1}^{T_2} \left( c + u + \ln(1+u) - au - b \right)^2 du .$$

This quantity will be minimized if the following two equations hold

$$\frac{\partial M}{\partial a} = - 2 \int_{T_1}^{T_2} u \left( c + u + \ln(1+u) - au - b \right) du = 0$$

$$\frac{\partial M}{\partial b} = - 2 \int_{T_1}^{T_2} \left( c + u + \ln(1+u) - au - b \right) du = 0$$

and  $T_2 > T_1$  .

After carrying out the indicated operations and solving for  $a$  and  $b$ ,



$$a = \left( 2c_2 - c_1 (T_1 + T_2) \right) / \left( (T_2^3 - T_1^3) / 6 + (T_1 T_2)(T_2 - T_1) / 2 \right)$$

$$b = \left( c_1 - a (T_2^2 - T_1^2) / 2 \right) / (T_2 - T_1)$$

where

$$c_1 = \left( c - 1 + .5 \cdot (T_2 + T_1) \right) (T_2 - T_1) + (T_2 + 1) \ln (T_2 + 1) \\ - (T_1 + 1) \log (T_1 + 1)$$

$$c_2 = c (T_2^2 - T_1^2) / 2 + (T_2^3 - T_1^3) / 3 \\ + \left( (T_2^2 - 1) \ln (T_2 + 1) - (T_1^2 - 1) \ln (T_1 + 1) \right) / 2 \\ - \left( (T_2 + 1)(T_2 - 3) - (T_1 + 1)(T_1 - 3) \right) / 4 .$$

Figure B-1 shows both the actual log likelihood ratio and the least square error approximation for a typical design signal-to-noise ratio. For the range 1.0 to 16.0 the agreement is good. Noise samples would be expected to be in this range 64% of the time and signal-plus-noise samples with the average S/N\* of 9 dB 99% of the time.

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\*The ratio S/N is measured after correlation but prior to envelope detection. For the case at hand 9 dB at this point corresponds to a -8 dB input S/N and a 12.2 dB output S/N.

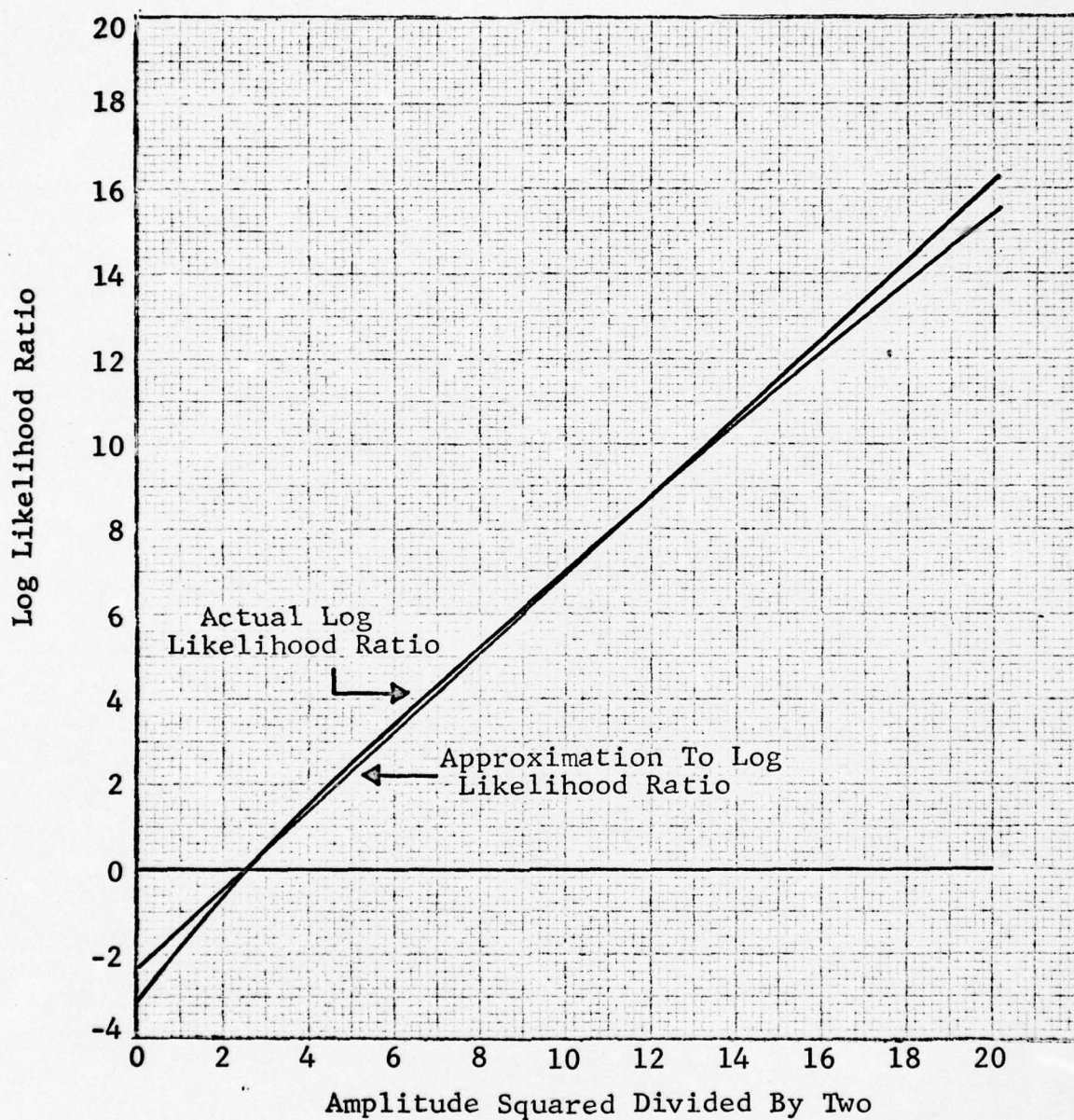


FIG. B-1 - COMPARISON OF ACUTAL LOG LIKELIHOOD RATIO TO LEAST SQUARE APPROXIMATION (U)





### B.3 PERFORMANCE EVALUATION

Tests were conducted to determine the performance of low-Doppler SLR processor and to compare it to a previously used low-Doppler processor. The criteria used in the comparison were the average track intensity and number of status units required. It was found that the four degree of freedom, chi-square model compared favorably to the SLR processor based on Rayleigh-Rice statistics.

The procedure used in deriving the intensity level threshold was the same as that used in the evaluation of the high-Doppler SLR processors and explained in detail in Appendix A, except the low-Doppler SLR operated on the noise samples drawn from an appropriate distribution. Several values of S/N were considered. At each ratio 40 different tracks widely separated in range and bearing were used to determine an average track intensity.

Figures B-2 through B-4 give the average track intensity as a function of ping numbers and for different values of S/N. The last figure shows only the theoretically predicted average intensity. Figure B-5 shows the contrast between the performance of the low-Doppler SLR based on Rayleigh-Rice statistics and the predicted performance of the four degree of freedom, chi-square model when the output signal-to-noise ratios are matched. The figure shows there is no significant difference. Figure B-6 compares the number of status units required in similar noise runs. Again no significant difference between the processor requirements are seen.

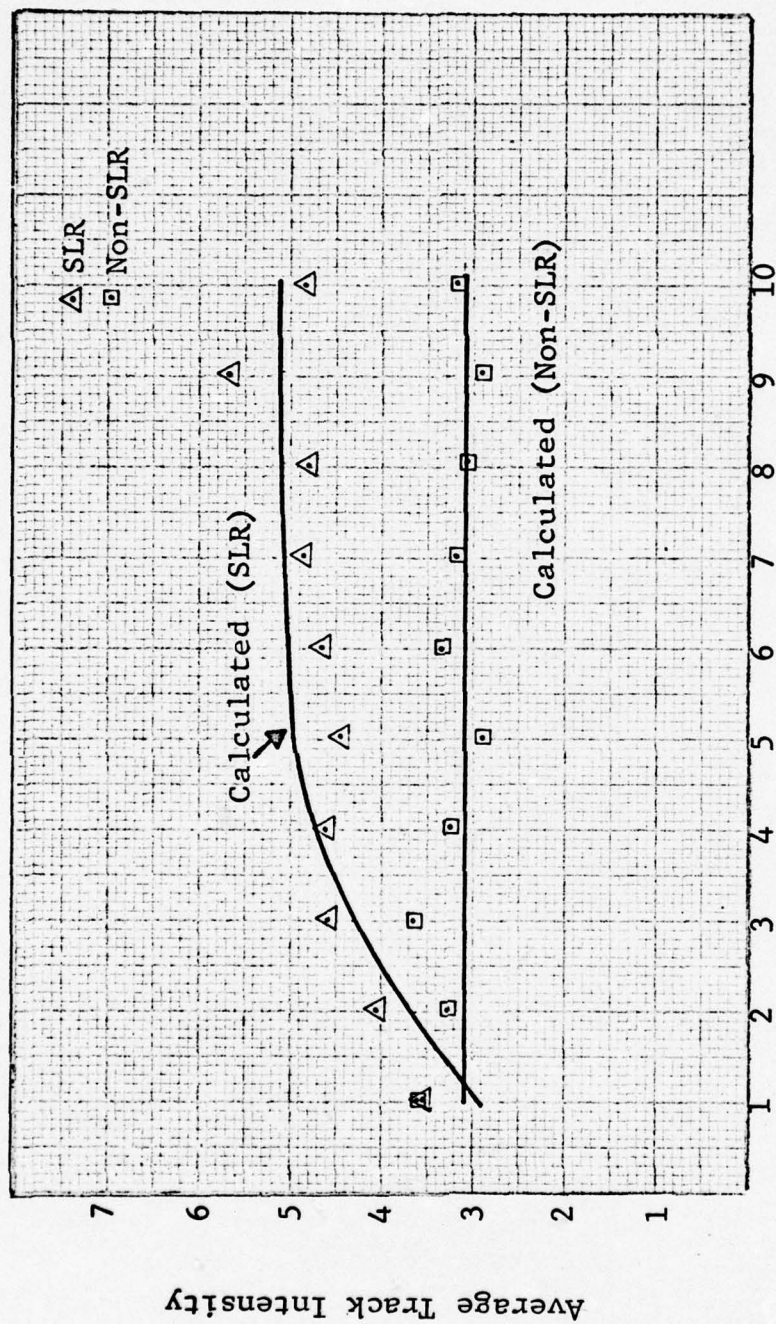


FIG. B-2 - AVERAGE TRACK INTENSITY FOR LOW-DOPPLER SLR TRACK S/N = 9.0 dB,  
DESIGN S/N = 9.0 dB (EXPERIMENTAL RESULTS) (U)

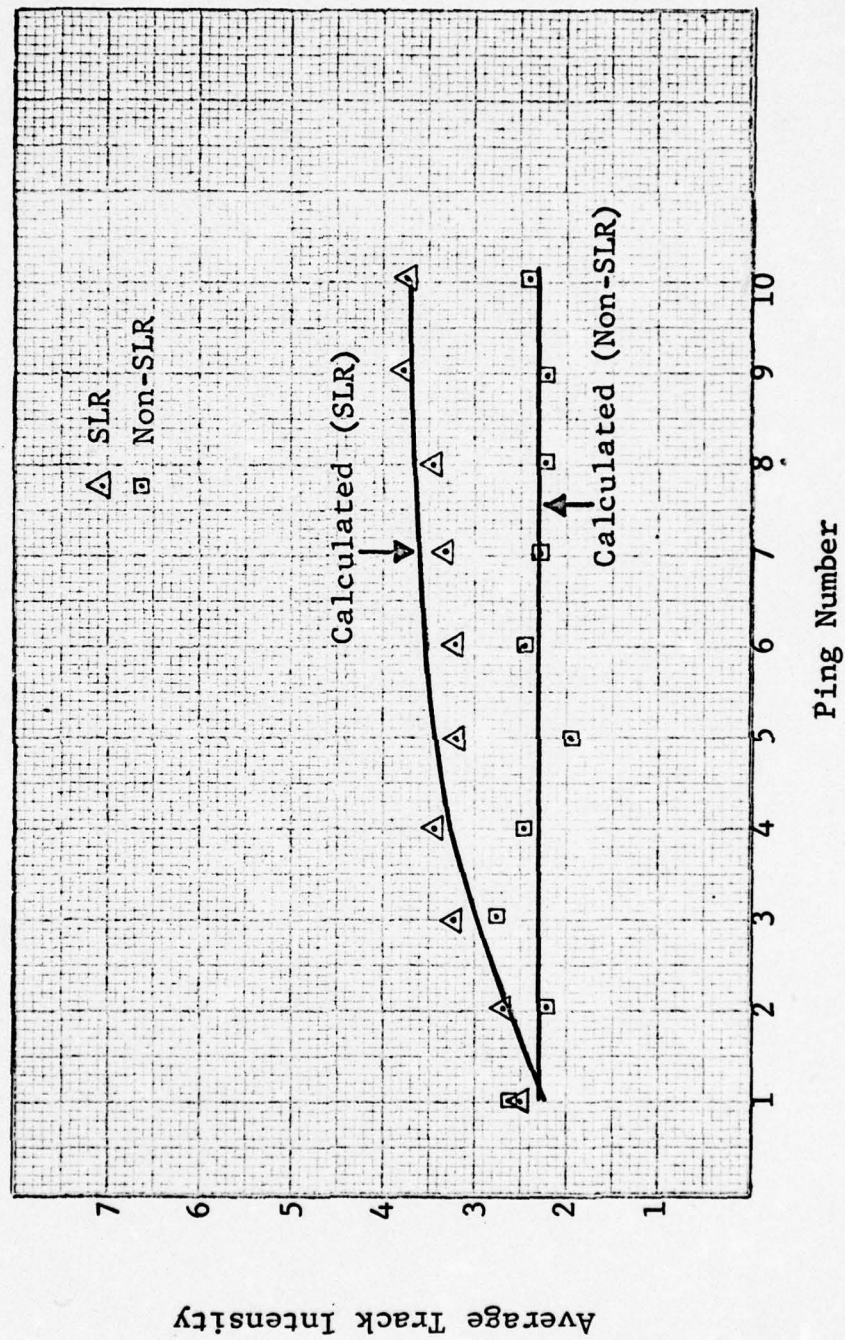
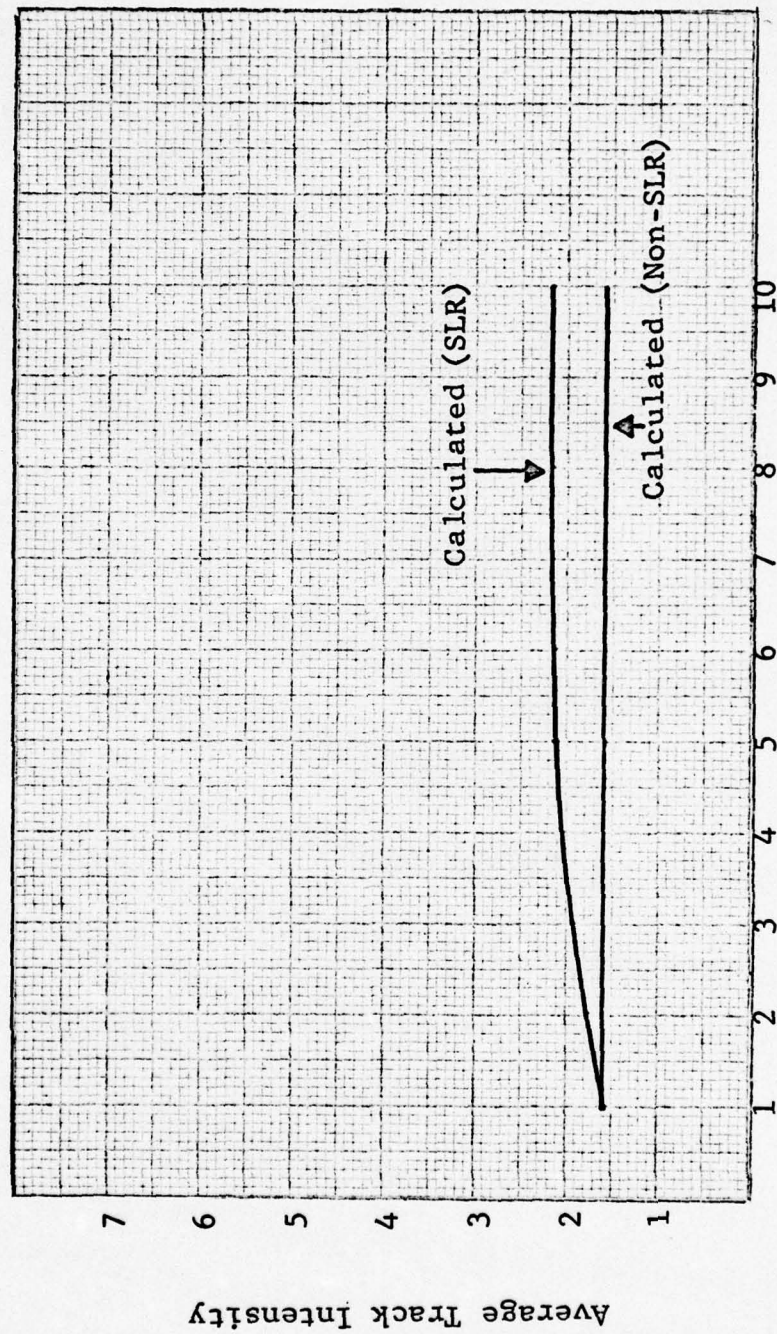


FIG. B-3 - AVERAGE TRACK INTENSITY FOR LOW-DOPPLER SLR TRACK  $S/N = 7.5$  dB,  
DESIGN  $S/N = 9.0$  dB (EXPERIMENTAL RESULTS) (U)





Ping Number

FIG. B-4 AVERAGE TRACK INTENSITY FOR LOW-DOPPLER SLR TRACK  $S/N = 6.1$  dB,  
DESIGN  $S/N = 9.0$  dB (U)

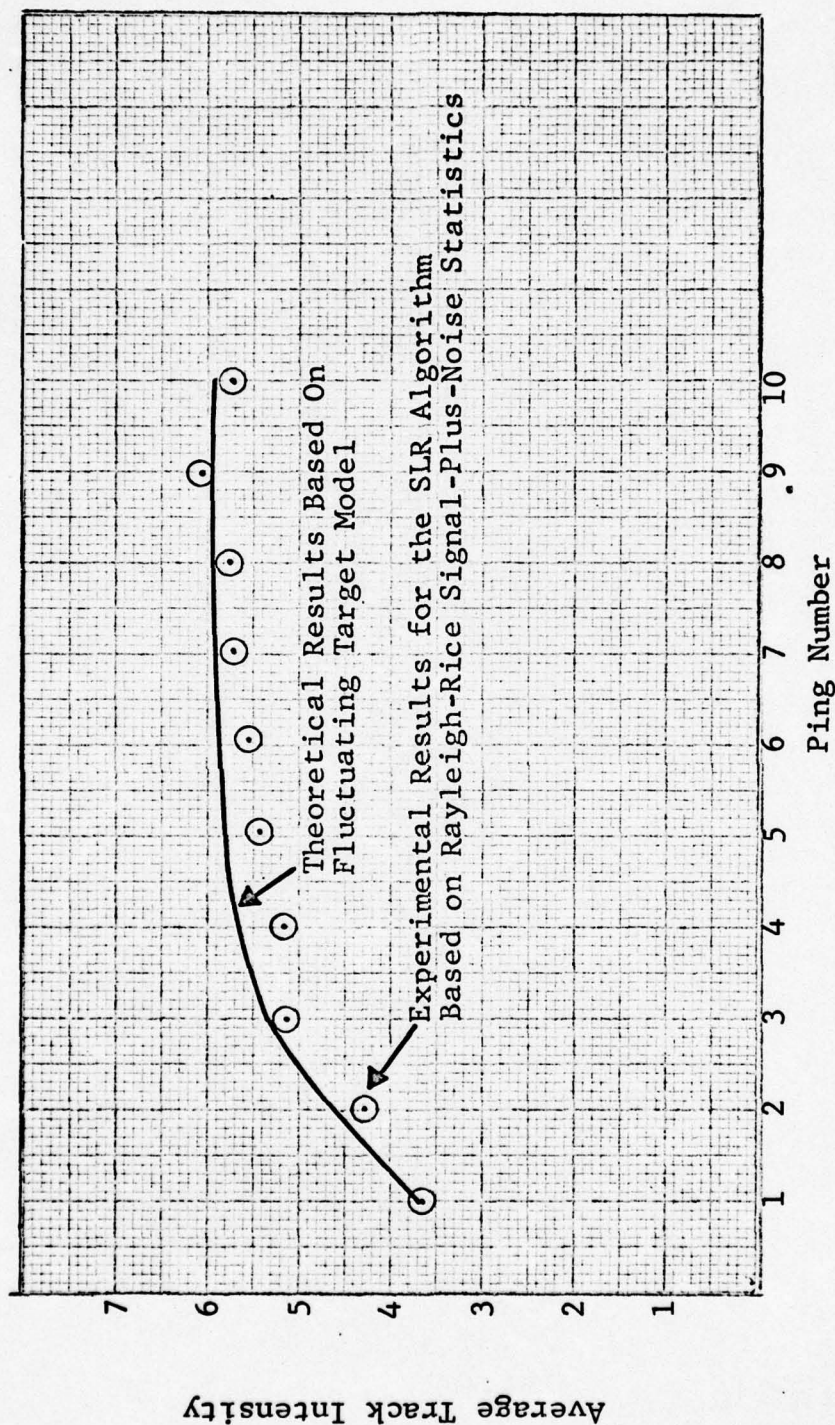


FIG. B-5 - AVERAGE TRACK INTENSITY FOR LOW-DOPPLER SLR TRACK  $S/N = 10.2$  dB,

DESIGN  $S/N = 9.0$  dB

⊙ ARE RAYLEIGH-RICE  $S/N = 12.0$  AND DESIGN  $S/N = 12.0$  dB (U)



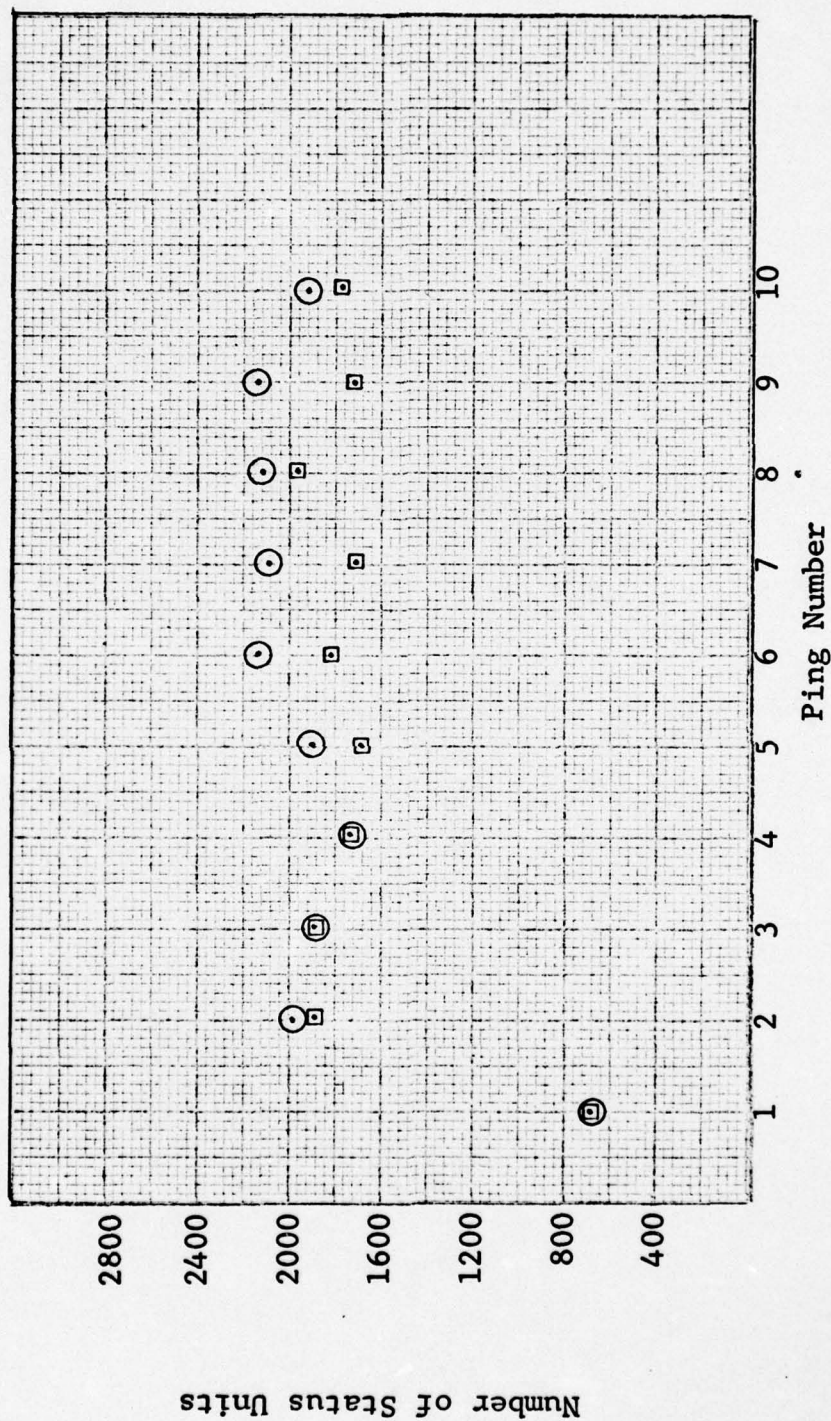


FIG. B-6 - COMPARISON OF NUMBER OF STATUS UNITS REQUIRED TO PROCESS NOISE ONLY DATE  
 FOR LOW-DOPPLER PROCESSOR, 4 d.f. MODEL (S/N)<sub>D</sub> = 9.0 dB (○),  
 RALEIGH-RICE MODEL (S/N)<sub>D</sub> = 12.0 dB (□) (U)





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## APPENDIX C

### DESCRIPTION OF THE MODEL FOR PREDICTING CLUTTER AND DETECTION PROBABILITIES IN THE SLR ALGORITHM



## C.1 INTRODUCTION

It is of constant interest to evaluate the sequential likelihood ratio processor under a variety of conditions and parameter changes. This may be accomplished by simulation techniques by processing actual data. However, in order to gain statistical significance a fairly large amount of data must be processed, requiring considerable computer time and expense. The quantitative answers obtained will necessarily reflect some statistical variation inherent in simulations. Hence, it is desirable to have a mathematical model to calculate the desired information. Such a model has been developed for the SLR processor. It is an extension of a previous model\* that solved a more limited problem. The limited model gave results that indicated trends in the SLR processor, but the present model gives quantitative results that compare favorably with the observed output data. Thus, by use of this model, the results of SLR processing may be predicted accurately and economically under a variety of conditions.

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\*Reeder (1970) op. cit., Appendix A.



## C.2 PROCESSING METHOD

The more limited model recognized only one particular kind of track, one that started on the first ping only. The present model allows three different tracks on any given ping cycle: Tracks starting on the present ping cycle, the previous ping cycle, and any tracks starting more than one ping cycle previously. Each of these three types of tracks are handled in a different manner by the SLR algorithm. The new model makes the same observation that the SLR processor deals with a linear sum of track amplitude samples and that the SLR probabilities may be determined by a convolution-type process that accounts for the SLR decision process.

For display purposes and, therefore, for determination of clutter marking and detection the largest likelihood ratio associated with a given range, bearing resolution cell is used. For tracks starting on the present ping,  $i$ , the probability that it is below a given threshold,  $T$ , is

$$\begin{aligned} P_M \left( L(x_i) \leq T \right) &= P_M (ax_i + b \leq T) \\ &= P_M \left( x_i \leq (T-b) / a \right) . \end{aligned} \tag{C-1}$$

This probability may be easily determined once the underlying probability distribution,  $F(x)$ , is specified. For tracks starting on ping  $i - 1$ , the probability of interest is





$$\Pr \left( \max L(x_{i-1}, x_i) \leq TL(x_i) \leq T \right) = P_i(\max x_{i-1}) + L(x_i) \quad (C-2)$$

$$- \ln N_L \leq TL(x_i) \leq T$$

where the  $\max x_{i-1}$  is taken over the  $N_L$  amplitude samples in the large tracking window. Note that the right hand side of Eq. (C-2) is true because there is only one sample  $x_i$  per resolution cell per ping. The probability distribution function of the largest amplitude sample may be easily determined by

$$F_{\max}(x) = \left( F(x) \right)^{N_L}$$

and then combined with the single ping probability distribution by convolution to determine required probability.

The last expression concerns the tracks that have some track history and therefore some observed range rate. The SLR processor defines a smaller track window,  $N_S$  samples, around the predicted to location. The probability of interest is

$$\Pr \left( \max L(x_k, x_{k+1}, \dots, x_i) \leq T, L(x_i) \leq T \right) \quad (C-3)$$

$$= \Pr \left( \max L(x_k, \dots, x_{i-1}) \right) + L(x_i) - \ln N_S \leq T, L(x_i) \leq T.$$



The probability of the maximum of the multiplying log likelihood is calculated in the same way as the maximum one ping log likelihood except that the cumulative probability distribution is different. This distribution, which is updated each ping cycle, is formed in the same way as the limited model\* was with the exception that the maximum is taken over the largest multiplying log likelihood ratio and the single ping log likelihood ratio before the convolution procedure. This is to allow for new tracks starting each ping cycle. Again after the maximum multiplying log likelihood ratio is determined the required probability is found by convolution.

Finally, the required probability is

$$\begin{aligned} \Pr(\max L(x) \leq T) &= \Pr(\max L(x_{i-1}, x_i) \leq T, \max(L(x_k, \dots, x_i) \leq T, L(x_i) \leq T)) \\ &= \Pr(\max(L(x_{i-1}, x_i)) \leq T, L(x_i) \leq T) \quad (C-4) \\ &\cdot \Pr(\max(L(x_k, \dots, x_i)) \leq T, L(x_i) \leq T). \end{aligned}$$

The last expression depends on the assumption that the log likelihood ratio for a track lasting two ping cycles is independent of the log likelihood ratio over more than two pings. This is not necessarily true since the two tracks may share the same track sample on ping  $i - 1$ ; however, tests indicated this assumption of independence does not introduce significant error.

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\*Reeder (1970), op. cit., Appendix A.



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The right-hand side of Eq. (C-4) is the product of the expressions of (C-2) and (C-3). While Eq. (C-1) does not occur explicitly, it is used in the convolution process to determine (C-2) and (C-3).

The model's ability to predict performance may be seen in Figs. A-1, A-2, A-3, A-4, B-2, B-3, and B-5, where observed and predicted values are shown.





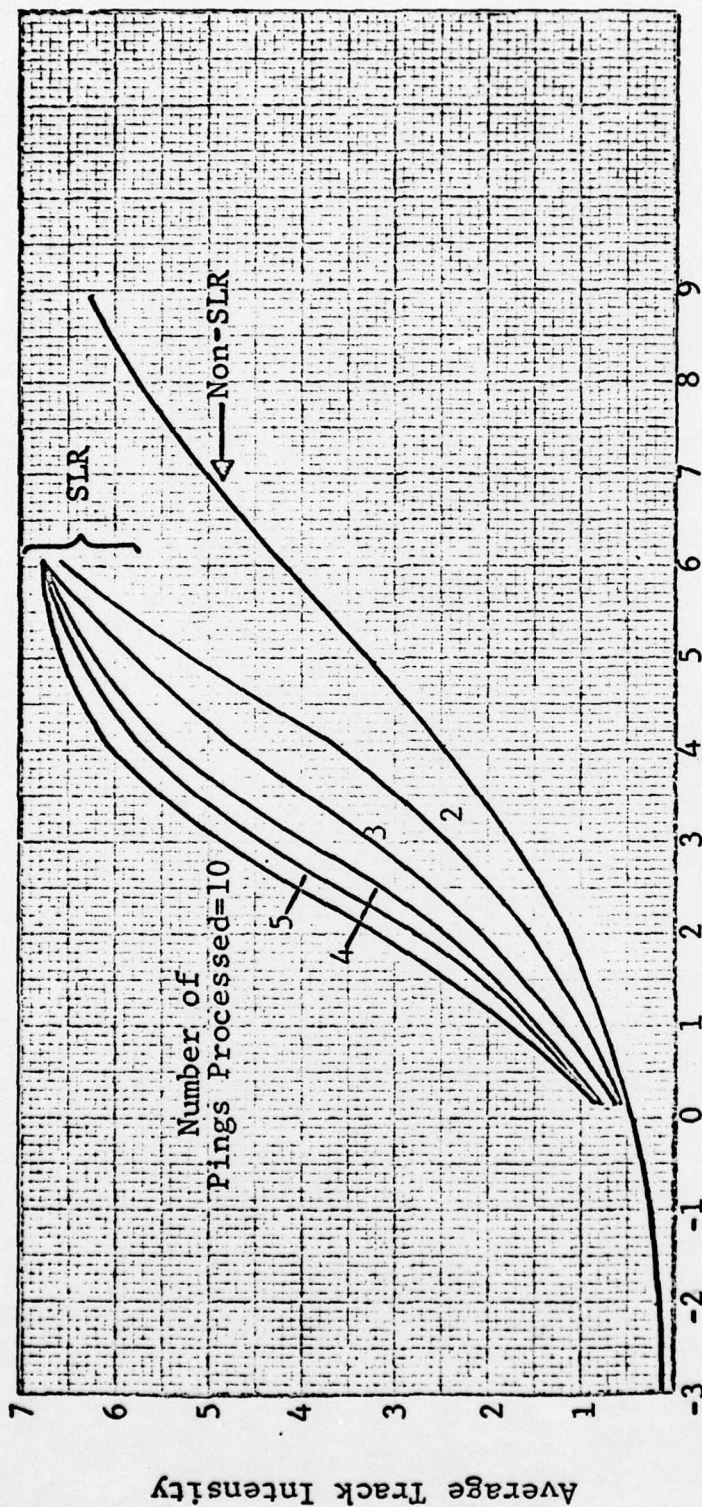
### C.3 A STUDY IN PARAMETER EFFECTS

In order to show the usefulness of the model and to further explore the high-Doppler SLR algorithm, the effects of certain parameter changes were calculated for the situation illustrated in Fig. 2 of the basic progress report. This figure shows the average track intensity for various input signal-to-noise ratios with normal tracking windows, the input data thresholded such that only 10% of the noise samples are considered by the SLR processor and a design S/N of 6.2 dB. Recall that Fig. 1 shows average track intensity for various input ratios of S/N for selected ping cycles after the commencement of the track (each curve is numbered with the number of pings elapsed). Figure C-1 shows that when the initial threshold is removed and the normal tracking windows are retained, there will be very little improvement; however, based on previous experience, computer storage requirements will increase considerably.\*

Figure C-2 shows the effect of no tracking windows (i.e., no target location uncertainty) and the 10% noise threshold. In this case improvement is noticeable in the low signal-to-noise ratio tracks. However, the assumption of no tracking window is tantamount to assuming perfect prior knowledge of the targets movements. The uncertainty in target maneuvers requires that tracking be performed and hence (in a sense) degrades the SLR processor's performance. A related case is shown in Fig. C-3 where no track window or initial threshold is assumed. Here there is a marked improvement in average track intensity. This case shows the maximum theoretical gain expected in the SLR with this design S/N.

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\*The present model, which considers only the maximum log likelihood ratio associated with each resolution cell, allows no prediction of computer requirements. This problem was considered briefly, but no promising approach to predicting computer requirements was found.

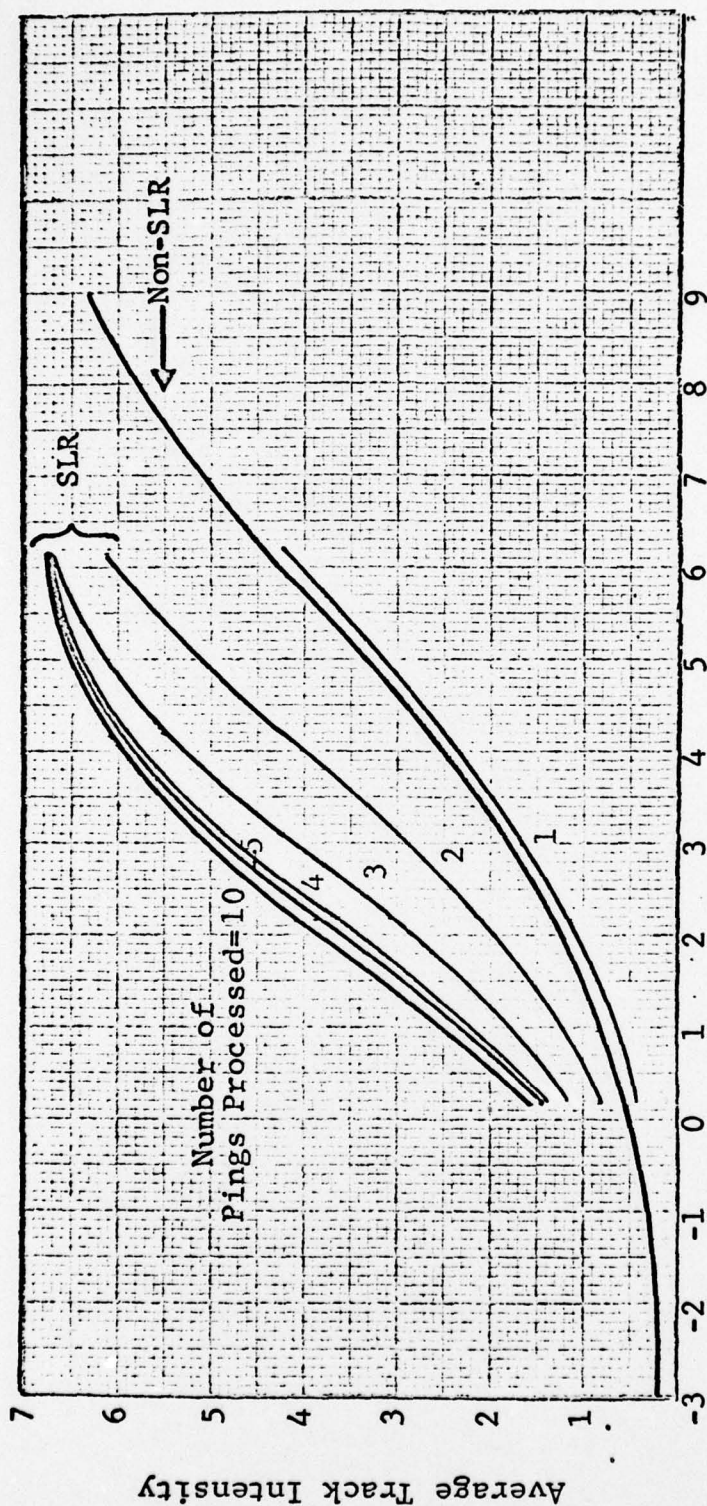


Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. C-1 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN S/N = 6.2 dB, NO INITIAL THRESHOLD (U)



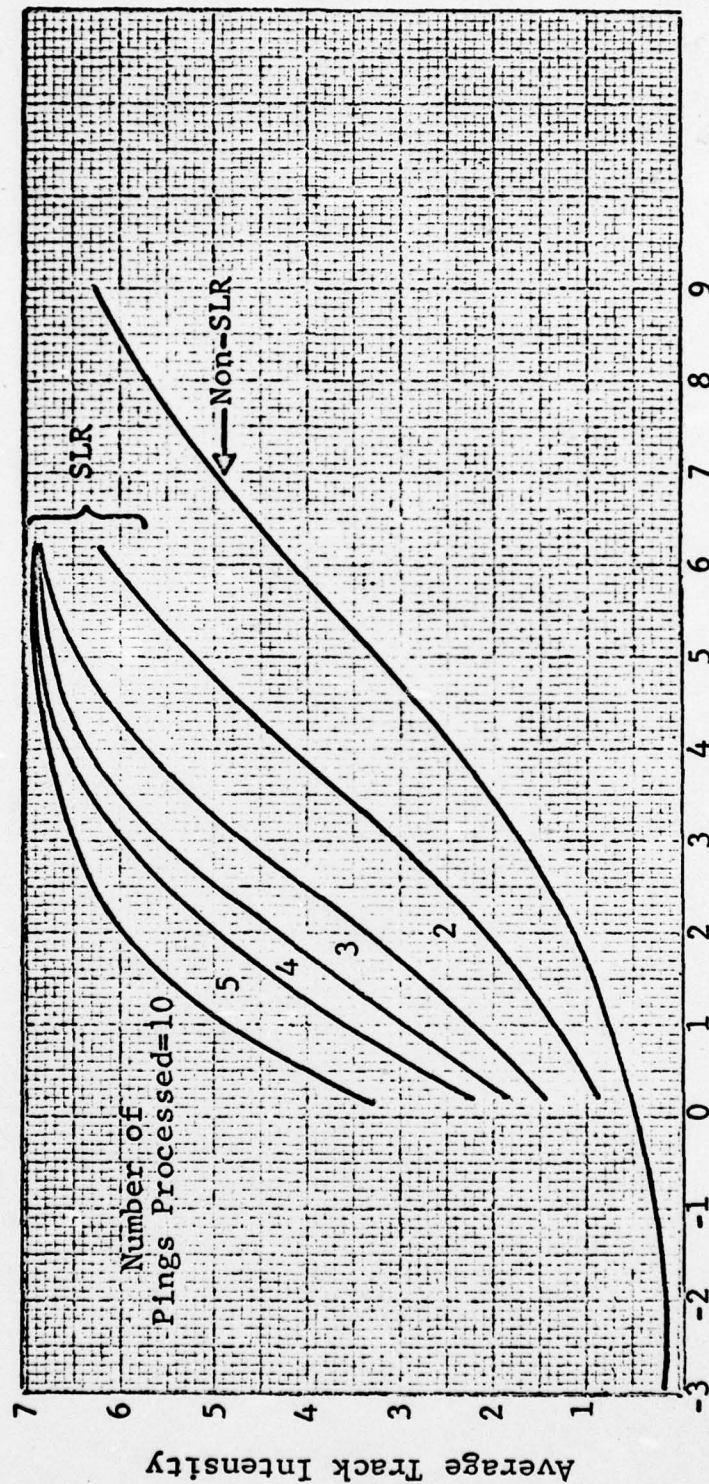


Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. C-2 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN  $S/N = 6.2$  dB, NO UNCERTAINTY IN TARGET LOCATION (U)





Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. C-3 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN S/N = 6.2 dB NO INITIAL THRESHOLD

NO UNCERTAINTY IN TARGET LOCATION (U)



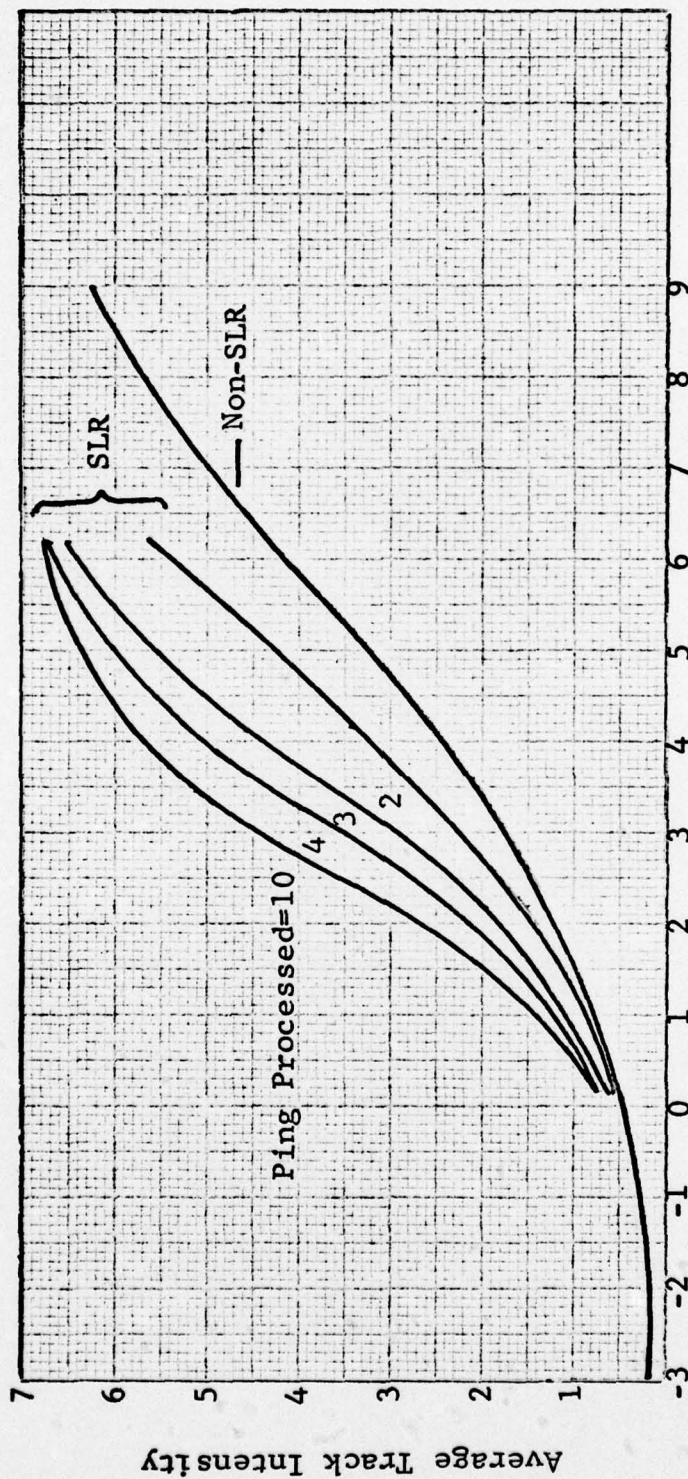
The design signal-to-ratio was lowered to 0.2 dB and the average track intensity calculated using normal tracking windows and threshold, and the results are presented in Fig. C-4. There is not a great deal of improvement in track intensities because of the correction for multiple tracks and initial thresholding. If no tracking window is used, Fig. C-5 shows improvement for lower ratios of S/N. The case of no initial threshold is not presented. By previous experience,\* it is known that significantly lowering the design signal-to-noise ratio greatly increase the computer requirements. Removing the initial threshold would only further aggravate an unreasonable situation.

The above has shown how the model for the SLR performance may be used to investigate the effect of parameter changes. Based on computer requirements, it does not seem reasonable to lower the initial threshold or the design signal-to-noise ratio. If the SLR processor's performance is to be improved for lower input signal-to-noise ratios, then the improvement must come from the tracking algorithm. Some reasonable way should be found to make the SLR processor more selective in its linkages. One method that bears investigation is to hypothesize that a signal-plus-noise track is consistent in all dimensions but has errors based on a multidimensional Gaussian distribution. This error distribution would be used to modify the joint log likelihood ratio based on the error between the observed track resolution cell and the predicted one. In essence small errors would result in small degradation while large errors would result in very large degradations. This contrasts with the equal degradation for all linkages in a in a specified multidimensional box in present SLR processor.

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\*H. A. Reeder, "Estimation of Computer Requirements," 1968.



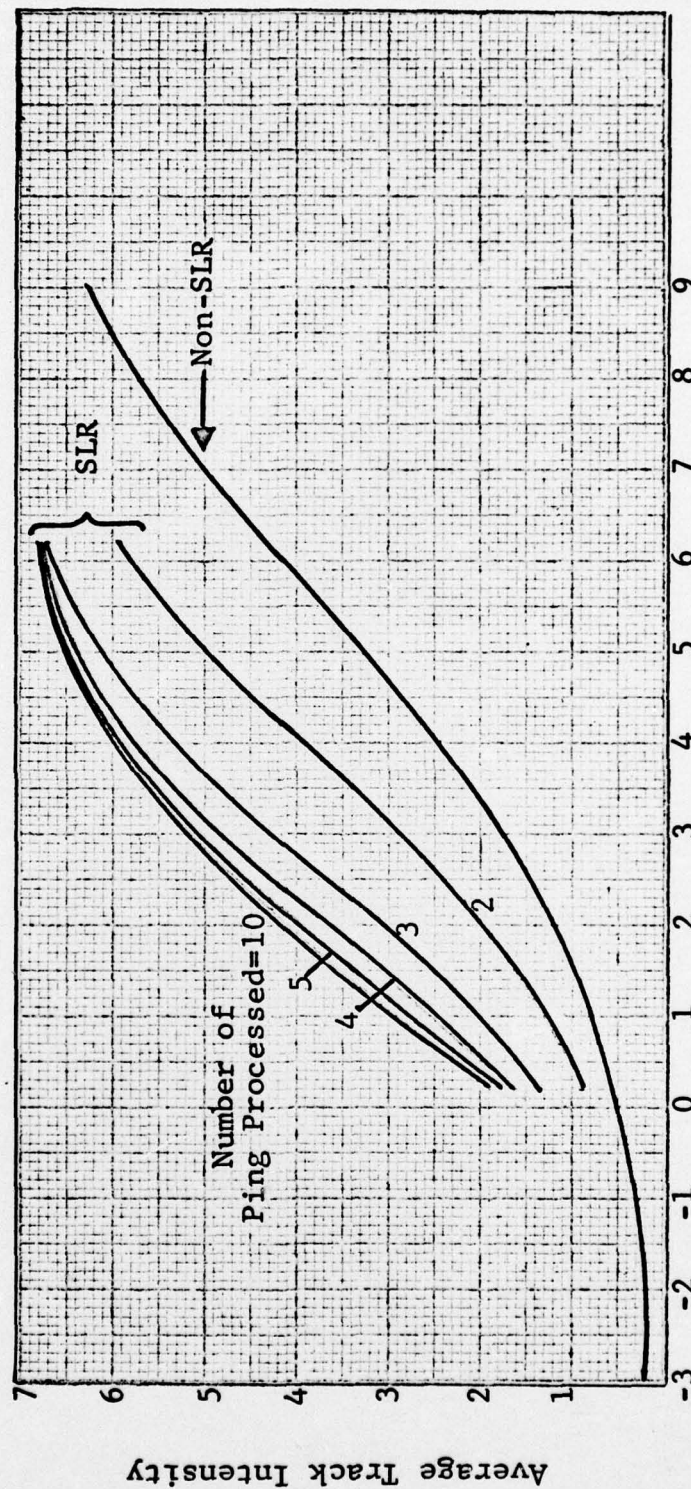


Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. C-4 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN S/N = 0.2 dB (U)





Signal-To-Noise Ratio At Output Of Combined Filter Bank, dB

FIG. C-5 - CALCULATED AVERAGE TRACK INTENSITY FOR CW SLR

DESIGN  $S/N = 0.2$  dB, NO WINDOW (U)



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A special version of this method was implemented and a short run carried out. Based on this limited data, it appears that this approach may have merit. However, many questions must be considered before this approach is adopted. For example, what is the effect when the target is rapidly maneuvering? As time and money permit, this new procedure will be considered.